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POLYTECHNIQUE MONTRÉAL
affiliée à l'Université de Montréal

Optimisation de la planification à court et moyen terme dans les mines souterraines

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Thèse présentée en vue de l'obtention du diplôme de *Philosophiæ Doctor*
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Optimisation de la planification à court et moyen terme dans les mines souterraines

présentée par **Louis-Pierre CAMPEAU**
en vue de l'obtention du diplôme de *Philosophiæ Doctor*
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RÉSUMÉ

La présente thèse s'inscrit dans le mouvement de numérisation des mines souterraines en s'attaquant au problème de planification. L'objectif global de la thèse est de fournir un outil d'optimisation des planifications à court et moyen terme permettant l'accès rapide à une solution optimale. La planification dans les mines souterraines pour ces horizons de temps est un problème difficile pour plusieurs raisons, notamment de par le grand nombre de ressources nécessaires, le grand nombre d'endroits de travail, les implications à long terme difficiles à prévoir et le niveau de précision requis. De manière plus spécifique, les objectifs de recherche sont de développer un modèle de programmation mathématique à court terme, un autre à court et moyen terme et un dernier en programmation par contraintes pour le court et moyen terme et de comparer ensuite les différentes approches.

La revue de la littérature disponible sur le sujet montre que la majorité des travaux sur la planification minière portent sur les mines en fosses. Bien qu'elles aient certaines ressemblances, les mines en fosse et les mines souterraines sont malgré tout trop différentes pour simplement appliquer les solutions de l'une à l'autre. On constate d'ailleurs cette disparité dans la différence entre l'offre commerciale de produits d'optimisation pour les deux types de mines. Au sein de la littérature portant sur le souterrain, la majorité des publications portent sur la planification à long terme. Quelques modèles sont disponibles pour les horizons de temps à court et moyen terme, mais sont spécifiques à certaines mines. De cette littérature, l'ensemble des modèles est basé sur la programmation mathématique, à l'exception d'un modèle de planification en temps réel, mais qui constitue un problème différent de celui présenté ici.

Un premier modèle de planification à court terme est présenté avec pour fonction objectif de maximiser les tonnes extraites tout en gardant un minimum de production de minerai pour chaque période de temps. Les variables utilisées pour la planification représentent des périodes d'une semaine et le modèle peut être résolu pour des exmeplaires allant jusqu'à six mois. Plusieurs tests sont effectués sur des données inspirées d'une mine canadienne et une analyse détaillée des solutions montre la grande différence entre la solution de la relaxation linéaire et le problème entier. Un exemple d'application réel est ensuite démontré afin de fournir les explications sur comment le modèle serait appliqué dans un tel contexte.

Un deuxième modèle en programmation mathématique est présenté pour la planification intégrée à court et moyen terme. Les variables de planification y représentent des périodes d'une semaine pour les trois premiers mois de planification et des périodes de trois mois pour les suivantes. Un premier objectif consiste à maximiser la valeur actuelle nette des activités planifiées, mais un second est aussi présenté où la valeur absolue de la valeur actuelle nette est maximisée. Il est démontré que le deuxième objectif permet une meilleure utilisation des ressources tout en conservant le même niveau de production, et correspond mieux à ce qui serait implémenté en un contexte réel. De plus, on démontre que la relaxation linéaire de ce dernier est beaucoup plus près de la solution entière, facilitant ainsi la résolution du problème. Un exemple d'application à des scénarios réaliste est ensuite présenté pour fournir un cadre d'application au modèle et les avantages de la planification à court et moyen terme intégré sont présentés.

Un troisième modèle est ensuite introduit, celui-ci utilisant la programmation par contraintes. L'objectif utilisé est de maximiser la valeur actuelle nette des activités. Le choix de ce dernier est fait afin de fournir une base de comparaison connue pour les modèles de programmation mathématique et de programmation par contraintes. Les résultats démontrent que ce nouveau modèle permet de résoudre avec une précision au quart de travail des exemplaires de plus d'un an. Une adaptation du modèle précédent permet de démontrer qu'aucune des exemplaires ne peut être résolue par celui-ci à ce niveau de précision et pour tel horizon de planification.

La thèse se conclut en présentant quelques travaux en cours comme le développement d'un modèle de planification en temps réel et une adaptation du modèle de programmation par contraintes à un problème de mine en fosse. L'inclusion de l'aspect stochastique dans le modèle est finalement discutée ainsi que le potentiel d'une application réelle à une mine en production.

ABSTRACT

This thesis is part of the current trend of digitization in underground mines by addressing the problem of mine planning. The overall objective of the thesis is to provide a tool for short and medium-term optimization of plannings, allowing optimal solutions to be found in a short time. Underground mine planning for these time horizons is a difficult problem for a number of reasons, including the number of resources required, the large number of work places, the long-term implications of short-term decisions and the level of accuracy required. More specifically, the research objectives are to develop a mathematical programming model for short-term, another for short- and medium-term and a last one using constraint programming for the short- and medium-term and then compare the different approaches.

A review of the available literature shows that the majority of work in mine planning is about open-pit mines. Even though they have some similarities, open-pit mines and underground mines are still too different to simply apply the solutions from one to the other. This discrepancy in the difference between the commercial offer of optimization products for both types of mines is another proof of this. Within the underground literature, the majority of publications focus on long-term planning. Some models are available for short- and medium-term time horizons, but are mine specific. From this literature, all the models are based on mathematical programming, with the exception of one real-time planning model, but it adresses a very different problem from the one presented here.

First a short-term planning model is presented with an objective function of maximizing tonnes mined while keeping a minimum of ore production for each time period. The planning variables represent one-week periods and the model can be solved for instances of up to six months. Several tests are carried out on data inspired by a Canadian mine and a detailed analysis of the solutions shows the large gap between the solution of the linear relaxation and the integer solution. An example of a real application is then shown to provide explanations of how the model would be applied in this context.

A second model using mathematical programming is presented for integrated short- and medium-term planning. The planning variables represent one-week periods for the first three months and three-month periods for the following ones. The first objective is to maximize the net present value of the planned activities, but a second one is to maximize the absolute value of the net present value. It is then shown that the second objective allows for a better use of the resources while keeping the same level of production, and better corresponds to what would be implemented in a real-life context. It is shown that the linear relaxation of

the latter is much closer to the integer solution, facilitating the resolution of the problem. An example of a realistic application to scenarios is then presented to provide a framework of application for the model and the benefits of an integrated short- and medium-term planning are presented.

A third model is introduced using constraint programming. The objective is to maximize the net present value of the activities planned. The choice of objective is made in order to provide a known basis of comparison for mathematical programming and constraint programming models. The results show that this new model allows to solve instances of more than one year at a precision of a work shift. A modification of the previous model shows that none of the instances can be solved using the mathematical programming model at this level of precision and this planning horizon.

The thesis concludes by presenting some work in progress including the development of a real-time planning model and the modification of the constraint programming model so that it can be applied to an open-pit mine problem. The inclusion of the stochastic aspect in the model is finally discussed as well as the potential for an application to a mine in production.

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CHAPITRE 1 INTRODUCTION

Depuis quelques années, on assiste à la numérisation de l'industrie minière. Bien que ce changement ait été opéré dans plusieurs autres domaines depuis de nombreuses années, dans le domaine manufacturier notamment, l'industrie minière reste en retard sur ce point. Ernst & Young identifie d'ailleurs dans une analyse récente, voir [1], le manque d'efficacité numérique comme le deuxième plus important risque pour l'industrie minière lors des deux prochaines années. Au sein même de l'industrie, on remarque que la plupart des efforts de numérisation existants se trouvent au niveau des mines en fosse comme en témoigne la vaste gamme de produits d'optimisation disponibles pour celles-ci. Parmi les raisons possibles de ce débalancement, on trouve sans doute le grand nombre de spécificités propres à chaque mine souterraine comme les méthodes de minage, le type d'accès et le type de transport de minerai. La thèse présentée ici s'inscrit dans ce mouvement de numérisation des mines souterraines en développant une approche permettant l'optimisation de la planification à court et moyen terme dans les mines souterraines. Nous présenterons ici avant tout une brève description de certains termes et concept nécessaires à la compréhension du présent document.

1.1 Définitions et concepts de base

Le développement d'une mine souterraine est un processus s'étalant sur plusieurs dizaines d'années et marqué par une grande incertitude. Cette incertitude vient principalement du fait que la forme et la teneur du gisement à exploiter doivent être estimées à partir d'une quantité limitée d'information. Les propriétés du gisement ne sont véritablement connues que lorsque celui-ci est extrait et traité. De plus, chaque information supplémentaire est extrêmement coûteuse, que ce soit en temps de forage dans une région éloignée lors des développements préliminaires ou pour le développement d'une galerie d'exploration une fois en production. Cette quantité d'information limitée a pour effet de rendre la planification des activités d'une mine très variable à travers le temps. Pour cette raison, les activités les plus éloignées en termes de temps de la planification sont généralement planifiées avec très peu de détails puisqu'ils sont appelés à changer.

1.1.1 Planification minière souterraine

La planification des opérations d'une mine se fait traditionnellement de manière séquentielle c'est-à-dire que plusieurs niveaux de planification sont réalisés indépendamment les uns des autres et de plus en plus détaillés, où la planification d'un niveau sert de paramètres initiaux au niveau suivant. Premièrement, une planification à long terme est réalisée afin de produire une planification globale pour l'ensemble de la vie de la mine. Comme celle-ci peut s'étirer sur plusieurs dizaines d'années et inclure plusieurs dizaines de milliers de tâches, des périodes d'un an sont généralement utilisées et plusieurs tâches sont agglomérées afin de former une seule tâche représentative de l'ensemble. Par exemple, au lieu de planifier l'extraction de chaque chantier inclus dans une veine, on agglomérera l'ensemble des chantiers en une seule grande tâche. En plus de réduire la taille du problème de planification, cette approche traduit le fait que de manière générale tout chantier ou veine dont l'extraction est prévue pour plus d'un an dans le futur n'est en fait qu'une estimation basée sur quelques trous de forage et est presque inévitablement appelée à changer de forme et propriétés au fur et à mesure que son exploitation se rapproche. Ce niveau de planification est généralement révisé tous les ans, afin de prendre en compte les nouvelles informations collectées durant l'année et de refléter la progression réelle par rapport à la précédente planification. À ce stade, l'objectif principal de la planification consiste surtout à maximiser la valeur monétaire de l'exploitation sur l'ensemble de la vie de la mine.

La première année de planification à long terme est ensuite détaillée en planification à moyen terme. Ainsi, la première année est découpée en périodes variant généralement de un à trois mois afin d'obtenir une estimation plus précise de la séquence d'activités. À ce niveau, plus d'informations sont disponibles sur la forme et les caractéristiques de la partie de gisement à exploiter et une planification plus fidèle à la réalité d'exploitation est produite. L'objectif du planificateur devient plutôt d'organiser globalement les tâches de manière à respecter les objectifs de production tirés de la planification à long terme et sur lesquels la performance de la mine seront évaluées e.g. mètres de développement par mois, onces de métal extraits.

On prend ensuite la première période de cette planification à moyen terme pour la découper en une planification à court terme, avec des périodes allant typiquement d'une à deux semaines. À ce stade, la plupart des décisions ayant un fort impact sur la valeur du projet ont déjà été fixées par les niveaux précédents. L'objectif à ce point est donc similaire à celui à moyen terme, c'est-à-dire respecter les cibles établies par le niveau de planification supérieur, mais ici avec beaucoup plus de précision. Une plus grande précision peut être atteinte à ce niveau puisque plusieurs paramètres sont maintenant fixés par les planifications à moyen et long terme, laissant moins de possibilités. La planification à court terme est généralement révisée à une fréquence hebdomadaire ou bi-hebdomadaire. Finalement, sur une base quotidienne un horaire en temps réel est produit, assignant les équipements et équipes à la réalisation des tâches afin de respecter le plan à court terme ; l'objectif étant simplement d'affecter les ressources aux bonnes tâches afin de réaliser la planification à court terme dans les délais.

1.1.2 Dévelopement et production

On sépare généralement les activités d'une mine souterraine en deux catégories, soit développement et production. La première regroupe l'ensemble des activités permettant d'accéder aux zones minéralisées. Les développements sont en quelque sorte des dépenses nécessaires à la réalisation d'un profit et représentent un des aspects critiques de la vie d'une mine. À l'inverse, la production regroupe l'ensemble des tâches liées à l'extraction du minerai, et donc liées à un revenu. De par leur nature, les deux sont planifiés de manière très différente. Les tâches de production visent généralement à produire un maximum de minerai excavé pour un minimum de ressources, alors que les développements visent à maximiser l'avancement pour un minimum de roche excavée. Les développements ont aussi beaucoup plus de support géotechnique comme des travailleurs s'y trouvent beaucoup plus souvent et qu'ils ont généralement une durée de vie plus longue que les ouvertures de production. Selon la méthode de minage choisie, on utilise parfois du remblai pour remplir les vides laissés par la production afin de stabiliser les ouvertures. Ce remblai peut être sous plusieurs formes et contenir un pourcentage variable de ciment selon les propriétés recherchées. Il faut généralement trois semaines pour que le remblai se solidifie et qu'il soit sécuritaire de travailler à proximité.

1.1.3 Méthodes de minage

Les dimensions et configurations des travaux de développement sont dépendantes de la méthode de minage choisie, qui elle-même dépend de la forme et des caractéristiques du gisement. De nombreuses méthodes de minage existent et une description de celles-ci peut être trouvée dans [2]. Comme les données utilisées dans le cadre de cette recherche proviennent d'une mine utilisant les méthodes long trou et coupe et remblai, nous donnerons ici une brève description de ces deux méthodes. Il s'agit aussi des deux méthodes les plus utilisées au Québec dans les mines de roches dures souterraines.

La méthode de minage long trou est particulièrement propice à l'exploitation de gisement sous forme de veines minces à inclinaison élevé. Bien que de nombreuses variations existent, l'idée générale de la méthode est de premièrement développer des galeries, appelées sills, dans la veine minéralisée à intervalle régulier. La hauteur entre ces galeries est généralement limitée par les propriétés mécaniques de la roche en place. Une fois ces galeries développées, on envoie une foreuse de production forer des trous entre les différents niveaux. On effectue ensuite le sautage d'une portion de la veine et le minerai est collecté par des accès adjacents au sill du bas. On peut ensuite remblayer l'excavation produite ou non, dépendamment des choix d'ingénierie.

La méthode coupe et remblais s'applique elle aussi aux gisements relativement minces, mais est généralement conseillée lorsque la qualité de la roche ne permet pas l'excavation non supportée de grande ouverture. Elle consiste à développer une série de galeries au sein du gisement afin de l'exploiter en tranches successives. Ainsi, une première galerie est développée dans le sens de la minéralisation, pour être ensuite remblayée avant le début de l'excavation de la galerie adjacente. Si cette méthode est généralement considérée plus sécuritaire que la méthode long trou, elle ne permet cependant pas d'atteindre les mêmes taux de production que celle-ci.

On trouve une explication plus visuelle de ces deux méthodes de minage dans une série de vidéos produite par le producteur d'équipement Épiroc, anciennement Atlas Copco, qui permet de facilement visualiser les deux techniques (Voir [3] et [4]).

1.1.4 Équipes de travail

Plusieurs équipements spécialisés sont requis dans une mine souterraine et leurs différentes tâches sont reliées entre elles par un ensemble de précédences selon le type de galerie ou du type de production. Typiquement, le développement d'une galerie commence par le déblayage de la roche par la chargeuse navette, ou LHD, laissée par le sautage précédant. Une fois le déblayage complété, l'écaillage doit être complété avec des perches ou à l'aide d'une écailleuse pour s'assurer que toute roche instable soit enlevée du plan de travail et la boulonneuse peut installer le support de terrain nécéssaire. La chargeuse navette vient ensuite nettoyer les roches tombées et la foreuse de développement, ou jumbo, peut commencer le forage de la face de travail. Une fois le forage terminé, les trous peuvent être chargés d'explosifs manuellement ou à l'aide d'une chargeuse, et ensuite connectés au système de sautage. Une particularité des mines souterraines est que pour des raisons de sécurité, les sautages ne peuvent être réalisés qu'entre les quarts de travail lorsqu'aucun mineur ne se trouve sousterre. Ainsi, peu importe le moment dans le quart où ces activités sont complétées, le cycle ne peut recommencer qu'au quart suivant. C'est pourquoi les planificateurs ont comme règle générale de planifier trois faces de développement par équipement de forage, afin de s'assurer qu'une d'elles soit toujours disponible et que l'équipement de forage soit utilisé au maximum. Dépendamment des mines, tous ces équipements peuvent être opérés par la même équipe de mineurs pour une face donnée, ou par plusieurs équipes spécialisées opérant uniquement un type d'équipement et se déplaçant entre les faces.

Pour ce qui est de la production, la méthode coupe et remblais implique sensiblement le même ordonnancement de tâches que le développement d'une gelerie. La seule différence est qu'à la fin du développement d'une tranche ou niveau de minerai, un mur doit être construit à l'entrée de l'accès du niveau pour contenir le remblai qui y sera inséré par l'équipe de remblayage. Pour ce qui est des chantiers long trou, ils commencent généralement par le forage de trous servant à l'installation de câbles de support dans la roche environnante. Ces câbles sont ensuite cimentés en place, par une câbleuse. Une fois la stabilité assurée, la foreuse long trou peut forer les trous de production au travers du gisement, ce qui peut durer plusieurs jours, voir plusieurs semaines. Une chargeuse de production est ensuite appelée à charger d'explosifs les trous forés et à les relier au système de sautage. Suivant le sautage, une chargeuse navette est assignée à déblayer le minerai sauté et une fois terminé, un mur est construit et on procède au remblayage lorsque nécessaire.

1.1.5 Mines en fosse et souterraines

La plupart des concepts décrits ici sont spécifiques aux mines souterraines. Comme il sera vu dans le chapitre suivant, plusieurs travaux portent sur la planification des mines en fosse, et beaucoup moins sont disponibles pour les mines souterraines. Malgré certaines ressemblances, les deux problèmes sont pourtant fondamentalement différents sur plusieurs aspects. La première différence concerne les sautages qui ne peuvent être réalisés qu'entre les quarts pour les mines souterraines, ce qui n'est pas le cas pour les mines en fosse. Deuxièmement, la différence de taille des sautages et des équipements de chargement fait que le nombre d'endroits de travail actifs simultanément est généralement beaucoup plus élevé dans une mine souterraine et la durée des tâches plus courte. Troisièmement, le milieu de travail confiné entraine des contraintes de congestion et de ventilation propres aux mines souterraines. Finalement, la structure de précédence est très différente d'une mine souterraine à une mine en fosse. Alors que les mines en fosse on généralement des teneurs plus basses, mais distribuées sur un plus grand volume, les mines souterraines elles impliquent de longues chaînes d'activités de développement afin d'avoir accès à une zone concentrée et limitée en volume de minerai.

1.2 Éléments de la problématique

La planification des activités d'une mine souterraine à court et moyen terme est un problème complexe pour de multiples raisons. Premièrement, beaucoup de ressources doivent être impliquées dans la planification, que ce soit au niveau des équipements, de la capacité de transport ou de la ventilation. Deuxièmement, un grand nombre de places de travail doivent être actives à tout moment afin de garder les équipements actifs, ce qui multiplie les possibilités d'assignations. Troisièmement, l'impact immédiat des activités planifiées est souvent difficile à visualiser puisque les développements nécessaires à l'exploitation d'une zone de production peuvent s'étendre sur plusieurs mois, voire même plusieurs années. Les planificateurs doivent donc prendre en compte les impacts que les choix de planification auront plusieurs mois à l'avance. Finalement, la planification doit être assez détaillée pour permettre de produire une planification réaliste et applicable considérant les nombreuses contraintes opérationnelles propres à une mine souterraine.

En plus de la complexité du problème, plusieurs raisons pratiques justifient l'importance du développement d'un modèle d'optimisation. Tout d'abord, les mines requièrent de manière générale d'énormes investissements initiaux et des coûts d'exploitation une fois en production tout aussi gigantesques. À titre d'exemple, citons la mine La Ronde, propriété d'Agnico Eagle, qui rapportait pour 2017 des coûts de production de 532$/once d'or pour une production totale de 348 870 onces, soit plus de 185 millions de dollars (voir [5]). La moindre amélioration dans ce genre de contexte permet de produire des économies intéressantes. Tel qu'expliqué précédemment, les planifications à court et moyen terme sont mises à jour de manière très fréquente. Ce processus est très chronophage et laisse peu de temps aux planificateurs pour optimiser chacune des itérations. Ce manque de temps pour optimiser mène à des planifications basées sur l'expérience passée des planificateurs qui appliquent des règles de décision non optimales pour établir leurs planifications. Finalement, dans un contexte de numérisation des mines, où des données de plus en plus volumineuses et fréquentes sont produites dans les exploitations souterraines, un outil mathématique est nécessaire afin de tirer profit au maximum de ces nouvelles informations.

La première approche envisagée a été de développer un modèle de planification à court terme permettant la prise en compte de toutes les contraintes propres à cet horizon de temps et de vérifier si ce modèle permettait de résoudre des instances de problème couvrant un horizon de planification à moyen terme. Lorsque ceci s'est montré impossible, un deuxième modèle a été développé afin de permettre une résolution de problèmes plus grands en terme d'horizon de temps considéré, au coût d'une précision réduite pour les périodes plus éloignées. Malgré tout, les plus grandes instances de notre ensemble de données ne pouvant toujours pas être résolues en un temps acceptable, il a été décidé de tester une approche de programmation par contraintes.

1.3 Objectifs de recherche

L'objectif du travail présenté est de créer un modèle et une méthode de résolution optimale et rapide du problème de planification des activités de minage pour les mines souterraines visant à optimiser la distribution des ressources et permettant la prise en compte de toutes les contraintes opérationnelles propres à ce milieu de travail et décrites dans les sections précédentes.

1.3.1 Objectifs spécifiques

1. Développer un modèle permettant une planification en détail des activités de minage à court terme.
2. Développer un modèle permettant une planification en détail des activités de minage à court et moyen terme.
3. Comparer les différentes approches de modélisation de la planification possible du problème.

1.4 Plan du mémoire

Nous présenterons dans les chapitres suivants l'ensemble des résultats et démarches menant à l'accomplissement de ces objectifs. Tout d'abord, une revue de la littérature existante sera présentée, incluant l'ensemble des sujets traités dans ce document. Un court chapitre présentera ensuite l'organisation globale de la thèse au lecteur afin qu'il puisse avoir une vue d'ensemble des travaux lors de la lecture de chacune des parties. Par la suite, une série de trois articles portant sur chacun des objectifs spécifiques constitueront les trois chapitres suivants. Enfin, une conclusion présentant une synthèse des travaux, leurs limitations ainsi qu'une discussion sur les travaux futurs conclura le document.

CHAPITRE 2 REVUE DE LITTÉRATURE

Une revue de la littérature complète de l'optimisation de la planification minière souterraine sera ici présentée afin de bien positionner les travaux effectués dans le contexte actuel de l'avancement de la recherche. Une brève revue des autres sujets abordés dans les chapitres suivants sera aussi présentée.

2.1 Ordonnancement de tâches

Le problème qui nous intéresse ici correspond à une des multiples variations des problèmes d'ordonnancement. Bien que plusieurs algorithmes et heuristiques permettent d'obtenir de bonnes solutions pour certaines de ces variations telles que détaillées dans [6], aucune de celles-ci ne correspond complètement à notre problème. En effet, celui-ci présente beaucoup de similitudes avec la catégorie de problèmes dits de séquençage de tâches ou communément "job shop" en anglais. Ce type de problèmes consiste à accomplir un ensemble de tâches données requérant l'intervention de différentes machines dans un ordre propre à chaque tâche et de manière à ce que la durée totale de l'exécution des tâches soit minimale. La plupart des problèmes de séquençage de tâches faisant partie des problèmes NP-difficiles, les différentes variations du problème font toujours l'objet de plusieurs recherches. Plusieurs revues des travaux effectués dans ce domaine existent telles que [7] et [8], ou [9] qui se concentrent sur les stratégies d'intelligence artificielle appliquées au séquençage de tâches.

Plus précisément, notre problème correspond à une variante de la catégorie des problèmes de gestion de projets avec contraintes de ressources dont les problèmes de séquençage de tâches ne sont qu'une application spécifique tel que démontré dans [10]. La définition de base de ce type de problèmes correspond à minimiser le temps d'exécution d'un ensemble de tâches liées entre elles par des liens de précédence, tout en considérant des contraintes de ressources renouvelables ou non renouvelables. On trouve dans [11] une revue des différentes variantes les plus populaires pour ce type de problème. Selon la classification utilisée dans cet article, notre problème ne correspond à aucune des catégories présentées, mais bien à un mélange de plusieurs de celles-ci e.g. objectif de valeur actuelle nette, délais d'exécution maximaux ou minimaux, selection de projet. La principale source de différence réside dans le fait que les modèles présentés ici sont libre de compléteter ou non certaines activitées, ce qui rend la résolution beaucoup plus complexe.

2.2 Optimisation dans les mines

L'optimisation dans les mines est un domaine qui existe depuis maintenant plusieurs dizaines d'années. À titre d'exemple, citons [12] qui présente une méthode exacte de résolution du problème de détermination du contour de fosse ultime (algorithme Lersch-Grossmann). Ce problème consiste simplement à déterminer pour une mine en fosse, quel ensemble de blocs respectant les contraintes de précédence d'extraction permettent l'obtention de la plus grande valeur. Cet ensemble de blocs peut ensuite servir à déterminer la forme finale, ou ultime, de la fosse à la fin de l'exploitation. Bien que l'algorithme Lersch-Grossmann fonctionne très bien pour ce problème, il est d'ailleurs encore utilisé dans certains logiciels de planification de mines en fosse, il ne permet pas de prendre en compte l'ordre d'extraction des blocs ainsi que les ressources nécessaires. C'est d'ailleurs pourquoi le sujet de la planification de l'extraction des blocs d'une mine en fosse fait toujours l'objet de plusieurs recherches, telles que recensés dans [13] ou dans [14] plus spécifiquement pour le court terme.

Plus généralement, on peut trouver une revue des applications de la recherche opérationnelle à l'industrie des ressources naturelles dans [15] et à la planification dans les mines dans [16]. Parmi les applications du domaine minier, on trouve plusieurs exemples de techniques d'optimisation de forme de chantiers, équivalent souterrain du problème de fosse ultime, tel que recensé dans [17]. Parmi les autres sujets d'intérêt, on trouve aussi beaucoup de travaux sur l'optimisation du passage d'une mine en fosse à une mine souterraine (voir [18] pour n'en citer qu'un). Finalement, plus près du sujet qui nous intéresse, on trouve aussi plusieurs travaux portant sur la planification dans les mines en fosse. Tel que mentionné précédemment, le sujet de la planification des activités d'extraction d'une fosse est le sujet de plusieurs études. Par contre, sa contrepartie souterraine a fait l'objet historiquement de beaucoup moins d'études. L'une des raisons de ce débalancement avancé par l'auteur de [19] dans sa revue des développements en optimisation de la planification des mines souterraines est que la planification souterraine est beaucoup plus complexe que la planification de fosse. On peut aussi d'ailleurs constater le manque de solutions fiables dans l'offre commerciale de logiciels de planification minière.

2.2.1 Options commerciales

Le tableau 2.1 présente une liste non exhaustive des principaux fournisseurs de logiciels de planification dans le domaine minier. Les deux premières colonnes indiquent le nom du produit offert permettant la planification à long et à court terme et les deux dernières, les produits permettant l'optimisation de la planification à long et court terme. La différence entre un produit de planification et d'optimisation est que le premier facilite la tâche de planification des activité par des outils de visualisation alors le second permet de produire automatiquement une planification optimale. À la vue du tableau, on constate aisément que bien que tous les fournisseurs offrent des solutions de planification à long et court terme, très peu offrent des produits d'optimisation à long terme, et aucun à court terme. Il est aussi intéressant de mentionner que plusieurs de ces fournisseurs offre maintenant aussi des solutions d'optimisation de forme de chantier, ce qui n'était pas le cas il y a cinq ans, preuve que la recherche dans ce domaine est arrivée à un niveau de maturité assez élevé.

2.3 Optimisation dans les mines souterraines

Pour ce qui est de la recherche en planification minière souterraine, nous ferons ici une revue de la littérature disponible depuis 2007 inclusivement. Le lecteur peut trouver dans [30] une revue des publications précédant cette date.

2.3.1 Long terme

La plupart des premiers travaux en optimisation de la planification minière souterraine se concentrèrent surtout sur la planification à long terme. Plusieurs raisons peuvent expliquer cette tendance, notamment le fait qu'à une échelle de temps d'un an et plus, plusieurs contraintes opérationnelles peuvent être simplifiées et que la granularité du problème peut être assez grossière, ce qui réduit grandement la difficulté liée à la résolution du problème. Malgré tout, ces modèles peuvent générer d'énormes bénéfices, tel qu'expliqué dans [31], qui décrit les bénéfices d'une approche intégrée dans la planification de la production de cuivre de plusieurs complexes miniers au Chili. On trouve ensuite dans [32] deux techniques d'agrégation afin de réduire la taille des modèles de planification à long terme. La première, basée sur la technique d'agglomération “K-means”, permet de grouper les unités de minage par localisation, et la deuxième permet de regrouper les activités de minage en fonction de la planification existante de la mine. Les deux techniques permettent d'obtenir pour un modèle de planification long terme de la minière chilienne CODELCO des réductions de temps de résolution de plus de 70%.

Toujours dans l'idée d'accélérer les méthodes de résolution existantes, l'article [33] présente deux algorithmes basés sur les temps de début au plus tôt et au plus tard afin de réduire la taille du problème de planification à long terme à la mine Kiruna en Suède. L'année suivante, [34] présente un algorithme de type “Greedy Randomized Adaptive Search Procedure” permettant aussi de résoudre un problème de planification à long terme de mine

Tableau 2.1 Aperçu de l'offre commerciale en logiciel de planification minière

	Planification		Optimisation	
	Long Terme	**Court Terme**	**Long Terme**	**Court Terme**
Dassault Systemes [20]	MineSched	MineSched	-	-
Datamine [21]	Studio UG	EPS	SOT	-
Deswik [22]	Sched	Sched	SOT	-
Hexagon Mining [23]	UG Pro	UG Pro	-	-
Maptek [24]	Gantt Scheduler	Gantt Scheduler	Gantt Scheduler	-
Micromine [25]	Scheduler	Scheduler	-	-
Minemax [26]	IGantt	IGantt	IGantt	-
MineRP [27]	Planner	EPS	-	-
Promine [28]	Mine Planning	Mine Planning	-	-
RPM Global [29]	XPAC	XACT	-	-

souterraine basée sur les contraintes opérationnelles propres à la méthode de minage par blocs foudroyés. [35] décrit ensuite une formulation dite classique et une autre améliorée permettant de réduire le temps de résolution du problème de planification sur des mines conceptuelles allant de 10 à 50 chantiers. La nouvelle formulation se base sur l'agglomération de variables liées par des liens de précédence pour réduire grandement le nombre de variables du problème. On retrouve dans [36] un modèle de planification de complexe minier composé de plusieurs mines souterraines et en fosse. Le modèle, utilisé par la minière CODELCO, permet notamment d'opérer les différents complexes miniers de manière optimale, mais aussi de simuler différents scénarios d'investissement. Au cours de la même année, [37] propose une heuristique basée sur la relaxation lagrangienne des contraintes de précédence et de ressources pour résoudre le problème. On trouve ensuite dans [38] une approche différente au problème d'optimisation à long terme avec un modèle permettant d'optimiser simultanément la teneur de coupure, soit la teneur en minerai minimale de la roche à extraire pour réaliser un profit, ainsi que la planification d'un complexe minier souterrain. [39] présente quelque temps après une modification du modèle permettant la prise en compte de l'aspect incertain de la teneur en place. Toujours en considérant l'aspect stochastique de la géologie minière, [40] propose un modèle stochastique de planification d'un complexe minier composé de plusieurs fosses et mines souterraines. Finalement [41] démontre les multiples applications possibles au domaine souterrain de la technique de résolution de relaxation linéaire expliquée dans [42] pour la planification de mine en fosse.

2.3.2 Moyen terme

Plusieurs publications se concentrent aussi sur la planification à moyen terme, où l'horizon de temps est généralement plus court et le niveau de précision plus élevé, avec la prise en compte de plus nombreux facteurs influencant la planification. [43] présente les modifications apportées à un modèle de planification à long terme afin de l'adapter à des horizons de planification d'un mois. Le modèle permet la distribution des ressources et des tâches tout en minimisant les écarts de production par rapport à la demande. [44] explique ensuite une heuristique permettant la résolution de problèmes réels provenant de la mine Kiruna basée sur l'agrégation des périodes de temps. Toujours appliquée à cette même mine, [45] présente une nouvelle heuristique permettant une résolution encore plus rapide basée cette fois sur des résolutions successives de sous-problèmes liés à differentes parties de la fonction objectif. Parmi les modèles qui permettent la prise en compte de plusieurs composantes simultanément, on trouve dans [46] un modèle d'optimisation de la forme et de la planification des chantiers d'une mine souterraine. [47] présente ensuite plusieurs formulations, dont une permettant l'optimisation de la teneur de coupure et de la planification des chantiers d'une mine souterraine à une précision mensuelle.

2.3.3 Court terme

Au niveau de la planification à court terme, l'un des premiers modèles dans la littérature est présenté dans [48]. Les auteurs proposent un modèle de programmation mixte en nombres entiers afin de planifier l'allocation des ressources pour les 120 prochains quarts de travail, soit environ 2 mois, pour les activités de production d'un groupe de chantier conceptuel incluant 50 chantiers. L'objectif du modèle consiste à minimiser la déviation de la production en tonnes par rapport à un tonnage prédéfini. Suivant cet article, les auteurs de [49] présentent un modèle intégré de planification à court et moyen terme appliqué à un groupe de tâches similaires. Le modèle est résolu pour des périodes de temps d'une semaine sur une durée de 48 semaines et vise encore une fois à minimiser les écarts entre la production prévue et réelle. On trouve ensuite un modèle présenté dans [50] qui permet la planification par semaine des deux dernières années de vie de la mine Lisheen en Irlande. Une heuristique aidant à la résolution de plus grands exemplaires du problème est finalement présentée dans [51] par les mêmes auteurs.

2.3.4 Temps réel

Les problèmes de planification en temps réel quant à eux sont assez différents des problèmes de planification à plus long terme. En effet, les problèmes de planification à ce niveau consistent essentiellement à attribuer des ressources spécifiques à des ensembles de tâches à réaliser, liées entre elles par des chaînes de précédence relativement courtes par rapport aux autres horizons de temps. De plus, l'objectif de ces modèles diffère comme on y cherche à compléter les activités planifiées dans un minimum de temps, plutôt que de trouver l'ordonnancement global des activités amenant le plus grand revenu ou tonnage. On trouve dans [52] une revue de la littérature disponible avant 2013 pour la planification en temps réel dans les mines souterraines. Depuis, les auteurs de [53] ont développé un modèle permettant de résoudre le problème de minimisation de la durée totale d'exécution des activités en temps réel pour le cas particulier d'une mine souterraine de potasse. Afin de résoudre les exemplaires les plus grands de leur problème, plusieurs techniques et heuristiques sont utilisées. Les techniques utilisées incluent le calcul de bornes inférieures et supérieures pour l'objectif basé sur les précédences et les durées des tâches, des réductions du domaine réalisable et la génération de solutions initiales basées sur des règles de décision. Plus récemment [54] présentent un modèle de programmation par contraintes pour résoudre un problème similaire basé sur une exploitation souterraine réelle. Ici encore, le modèle cherche à minimiser le temps d'exécution d'une série d'activités pour un horizon de temps de moins de 72 heures. Dernièrement, les auteurs de [55] ont publié une revue parallèle de la littérature en ordonnancement dans les mines souterraines et de la littérature en ordonnancement des tâches. Les auteurs y défendent le point que les problèmes d'ordonnancement de mine souterraine à très court terme i.e. une ou deux semaines, sont en fait des variations des modèles d'ordonnancement classiques et que la communauté scientifique gagnerait à s'en inspirer.

2.4 Programmation par contraintes

Finalement, la programmation par contraintes telle que mentionnée dans le chapitre 6 de la présente thèse a fait l'objet de très nombreuses études et applications. Le logiciel de résolution de programmation par contraintes utilisé fait d'ailleurs l'objet d'une publication (voir [56]) détaillant son lexique, fonctionnement et plusieurs applications possibles. Nous ferons ici une brève description des principaux éléments du processus de résolution.

La première phase de résolution consiste en une étape de prétraitement permettant une reformulation automatique de certaines erreurs courantes de formulation. Le choix de la méthode de formulation ayant un grand impact sur les performances du logiciel, ceci permet d'utiliser les méthodes de propagation de contraintes au mieux de leurs capacités. Par la suite, une propagation des contraintes permet d'établir un domaine réalisable initial pour chacune des variables. La propagation des contraintes se fait selon trois approches. La première utilise un réseau logique tel que décrit dans [57], qui permet d'établir les liens logiques entre les différentes variables binaires, comme les variables indiquant la présence d'une activité ou non. Ce réseau permet de détecter les infaisabilités dans les contraintes et de lier la présence de certaines contraintes à d'autres, permettant une meilleure propagation subséquente des contraintes.

Deuxièmement, un réseau temporel est aussi établi utilisant les contraintes de temps, comme celles de précédences par exemple, afin de réduire le domaine des débuts et fins possibles de chaque variable temporelle. On entend ici par variable temporelle toute variable représentant la présence d'un évènement à un temps donné. Le logiciel utilise une variation (voir [58]) du réseau temporel décrit dans [59], qui permet la prise en compte d'évènements dont la présence est incertaine, comme une tâche optionnelle par exemple. La propagation initiale dans le réseau est basée sur une amélioration de l'algorithme de Bellman-Ford présenté dans [60] et les propagations suivantes suivent la méthode décrite dans [61].

Troisièmement, les contraintes de ressources sont mises à profit dans la propagation des contraintes. Par défaut, le logiciel utilise une méthode de type «Timetabeling» pour propager les contraintes de ressources (voir [62]). Cette méthode consiste essentiellement à utiliser les débuts et fins au plus tôt et au plus tard de chaque activité nécessitant une ressource afin de calculer une utilisation minimale pour chaque période de temps. Cette information combinée à la capacité maximale de chaque ressource pour une période donnée permet de réduire le domaine réalisable de chacune des variables. Lorsque cette méthode ne permet pas de réduire assez le domaine réalisable, des algorithmes de types «Edge-Finding» plus coûteux en termes de calcul, mais généralement plus puissants, sont appliquées comme ceux présentés dans [63] ou [64]. Ces méthodes de propagation des contraintes (logique, temporelle ou de ressource) sont répétées lors de la résolution au fur et à mesure que de nouvelles informations sont découvertes et que le domaine réalisable est réduit.

Si la propagation des contraintes permet de réduire grandement le domaine réalisable, elle ne permet généralement pas de le réduire au point que l'identification de la solution optimale devienne triviale. C'est pourquoi le logiciel utilise un autre ensemble de techniques lorsque la solution optimale est recherchée comme c'est le cas dans cette thèse. La première technique appliquée, appelée «Large Neighborhood Search», est une modification de l'algorithme présenté dans [65] et décrite dans [66]. Il s'agit d'une heuristique partant d'une solution initiale réalisable explorant l'impact de certaines modifications sur la solution finale. Une relaxation linéaire du problème est aussi utilisée afin d'obtenir des débuts et fins probables pour chaque variable basée sur la fonction objectif. La procédure de relaxation est décrite dans [67]. Afin de prouver l'optimalitée des solutions trouvées à l'aide de ces algorithmes, on utilise en parallèle un algorithme appelé «Failure-directed search» décrit dans [68] qui permet de réduire l'espace de recherche de solutions en prouvant le plus rapidement possible l'infaisabilité de certaines branches. Cette combinaison d'algorithmes a été prouvée très efficace dans une série de test et comparaisons montrés dans [56].

Utilisant un autre logiciel, [69] présente six formulations d'une variation du problème de gestion de projets avec contraintes de ressources incluant des contraintes de calendrier ainsi que des intervalles maximaux et minimaux. Une comparaison entre ceux-ci et une formulation en MIP est ensuite présentée sur un ensemble de problèmes de référence. Dans le cadre d'une application réelle, [70] compare des méthodes de résolution basées sur la programmation par contraintes et la programmation en nombres entiers pour deux problèmes d'ordonnancement des tâches d'un robot mobile. L'article montre que la résolution par programmation par contraintes permet une résolution optimale plus rapide en moyenne que celle en nombres entiers pour le problème testé.

CHAPITRE 3 ORGANISATION DE LA THÈSE

Le chapitre 4 présente l'article "Short-term planning optimization model for underground mines", publié dans la revue Computers & Operations Research. On y présente un modèle de planification à court terme pour les mines souterraines. Dans ce premier modèle, contrairement aux modèles présentés dans les articles suivants, l'ensemble des équipes nécessaires aux tâches de production sont regroupées en une seule équipe et les endroits de travail ne sont pas sous-découpés en intersections. L'objectif du modèle consiste à maximiser le tonnage extrait tout en déviant le moins possible par rapport aux objectifs de production de minerai. Un taux d'actualisation est appliqué au tonnes extraites afin de favoriser l'extraction au plus tôt possible. Le modèle prend en compte toutes les contraintes opérationnelles propres à cette échelle de temps et permet la résolution d'exemplaires allant jusqu'à 24 semaines, pour des variables représentant la planification d'une semaine.

Le modèle est formulé de façon à ce que la préemption soit permise pour deux raisons principales. Premièrement, ceci permet la séparation de longues galeries de développement en plusieurs périodes de travail séparées, comme ce serait le cas en réalité. Deuxièmement, comme les tâches à planifier sont de durée très variable et avec comme dénominateur commun le quart de travail, soit 10 heures généralement, cette modélisation permet à des tâches précédente/successeur de finir et débuter au cours de la même semaine sans avoir à créer des variables pour chaque quart de travail. L'avantage d'une telle formulation est d'ailleurs démontré dans le chapitre. Cela se fait cependant au coût de rendre le problème plus complexe, comme la préemption crée beaucoup plus de solutions ou scénarios possibles. Outre ceci, l'article présente une analyse approfondie de la relaxation linéaire du problème, démontrant la piètre représentativité de la solution relaxée par rapport à la solution entière. Finalement, il y est démontré que l'inclusion d'objectifs de planification dans le modèle accélère grandement le temps de résolution du modèle, tout en étant plus représentative d'une utilisation en situation réelle.

Le chapitre 5 présente l'article "Integrated optimization of short- and medium-term planning in underground mines", soumis à la revue International Journal of Mining, Reclamation and Environment. Le premier modèle présenté au chapitre 4 étant limité à un horizon de six mois, les contraintes à moyen terme doivent y être imposées à l'aide de contraintes limitant les possibilités d'optimisation. L'article ayant démontré que cet horizon était généralement le plus long pouvant être planifié, le modèle ne permet pas d'englober complètement la première période de planification à long terme, soit un an. Cet article propose donc de faire une optimisation intégrée des planifications à court et moyen terme en s'inspirant de l'approche de planification utilisée par les planificateurs, i.e. produire une planification par semaine pour les prochaines activités et une planification par période de 3 mois pour les activités plus éloignées. Les données sont basées sur la même mise en situation, mais avec une plus grande précision au niveau des équipes de travail avec chaque long développement découpé en tâches et séparé pour chaque intersection. Même si le modèle est inspiré du précédent modèle, il existe tout de même plusieurs différences entre ceux-ci causées notamment par la différence de quantité de temps représentée par les variables et les horizons de la planification à moyen terme pouvant contenir de très nombreuses activités qui se succèdent.

L'objectif initial du modèle est de maximiser la valeur actuelle nette des activités planifiées. Ceci pose cependant problème pour deux raisons principales. Premièrement, malgré plusieurs tentatives et modifications, le modèle initial n'a pas pu être résolu à l'optimalité en des temps de calcul raisonnables. Parmi les essais faits, on trouve le développement d'heuristiques de type fenêtre glissante et autres, l'ajout de coupes, la relaxation progressive de contraintes, la réduction du problème par des débuts au plus tôt/fin au plus tard, la modification de la stratégie d'exploration de l'arbre de branchement, le changement de taille et nombre de variables de planification et l'optimisation des paramètres de résolution. Deuxièmement, les solutions trouvées ont tendance à ne pas utiliser les différents équipements au maximum de leur capacité. Elles se contentent de planifier le moins de développements possible pour aller chercher la production disponible. Des contraintes pourraient être imposées afin de forcer le modèle à utiliser les équipements à une certaine fraction de leur capacité, mais ceci entraine la difficulté de devoir fixer cette fraction. Il est en effet difficile de la définir puisque de par les relations de précédence, certains équipements ne peuvent tout simplement pas être assignés à certaines périodes de temps, ou le peuvent, mais seulement partiellement. L'ajout de telles contraintes complexifie aussi le problème.

L'utilisation de la valeur actuelle nette comme fonction objectif avait été initialement choisie pour sa popularité dans les modèles de planification à long terme, il a donc été proposé de modifier cet objectif afin de prendre la valeur absolue des coûts et revenus associés à chaque site. Ce changement a permis de régler le problème de temps de résolution puisque tel que démontré dans l'article, la relaxation de ce nouveau problème est beaucoup plus semblable à la solution finale. Il a aussi permis de régler le problème d'utilisation des équipements puisque les zones de production sont toujours développées le plus tôt possible, tout en gardant les équipes de développement actifs. Le coût des développements étant principalement dû à la main d'oeuvre, il peut être pratiquement considéré comme fixe dans un contexte réel où il faudrait payer pour ces ressources, qu'elles soient utilisées ou non. On peut définir le nouvel objectif comme la maximisation de l'exécution des activitées auquel on aurait appliqué une pondération en fonction de la valeur monétaire. Il s'agit donc d'un objectif similaire à celui du premier article mais pour lequel on aurait ajouter un poids plus grand aux tonnes de production qu'aux tonnes de développement. Les résultats de l'application des deux objectifs sont ensuite présentés ainsi que la comparaison des résultats d'une planification séparée par rapport à une planification intégrée.

Le chapitre 6 présente l'article "Short and medium-term optimization of underground mine planning using constraint programming" soumis à la revue Constraints. Il présente une nouvelle approche de résolution du problème de planification à court et moyen terme de mines souterraines en proposant une formulation basée sur la programmation par contraintes. Cette méthode de résolution ayant fait ses preuves sur des problèmes similaires, il a été jugé pertinent de comparer les différentes approches sur notre problème. Les tests présentés dans ce chapitre sont basés sur les mêmes données utilisées dans le chapitre 5. Les contraintes du problème sont aussi similaires à celles présentées au chapitre précédent dans leurs effets, mais formulées selon les pratiques de la programmation par contraintes. Il a été décidé d'utiliser l'objectif de la valeur actuelle nette plutôt que l'objectif modifié du précédent article puisque la valeur actuelle nette est plus reconnue et globalement utilisée, le but étant de comparer deux approches sur une base commune. Des tests non publiés ont cependant été faits et ont démontré que le changement d'objectif n'avait que très peu d'effet sur ce dernier modèle et donc, l'utilisation de l'un ou l'autre des objectifs serait envisageable dans une application réelle.

Comme le niveau de discrétisation du temps a généralement peu d'impact sur les temps de résolution de problèmes de planification en programmation par contraintes, l'unité de base de la planification a été réduite progressivement jusqu'à obtenir une planification par quart de travail tout en permettant des résolutions rapides et optimales du problème comme démontrés dans le chapitre. La comparaison avec une adaptation du modèle présenté au chapitre 5 pour permettre un niveau comparable de précision, soit une planification par quart plutôt que par semaine, prouve la supériorité du modèle de programmation par contraintes pour résoudre ce problème. Bien qu'une planification au quart de travail soit beaucoup trop précise pour une planification à moyen terme, considérant la grande incertitude liée aux activités plus éloignées dans le temps, elle est tout de même utilisée ici puisque les résultats montrent bien que cela ne rend pas le problème particulièrement plus difficile à résoudre.

CHAPITRE 4 ARTICLE 1: SHORT-TERM PLANNING OPTIMIZATION MODEL FOR UNDERGROUND MINES

Louis-Pierre Campeau, Michel Gamache, (2019), "Short-Term planning optimization model for underground mines", Computers & Operations Research
Disponible en ligne depuis le 16 février 2019

4.1 Abstract

Scheduling activities in an underground mine is a very complex task. Precedence relations, the great number of resources and the large number of work sites are some of the reasons for this complexity. This paper presents an optimization model for short-term planning that takes into consideration all parts of the development and production as well as specific limitations on equipment and workers. A preemptive mixed integer program is used in order to produce optimal planning over a short-term time horizon. Multiple tests made with various data sets and scenarios are then presented, including a comparison to a non-preemptive model and a case study.

4.2 Introduction

Mining projects are made possible through the investment of massive funds. Initial capital costs are huge, running costs are high and risk is higher than in most other businesses. Nevertheless, when managed efficiently, these projects can become very profitable. In order to reach profitability, effective planning is an essential and powerful tool for getting the most value out of a project. Activities throughout the life of the mine are planned with different levels of precision and time frames depending on the state of the project. This article presents a mathematical model that enables optimization and testing of different scenarios for short-term planning in an underground gold mine.

The objective behind the development of this model was to make a tool available to facilitate the transition from medium- to short-term planning by first enabling the testing of operational scenarios and assuring an optimal dispatch of resources. The reason why a mathematical model is needed is that the short-term planning of activities in an underground mine is too complex to be solved without the use of a decision support system due to the great number of interdependent decisions involved.

Several elements make this problem difficult. Even for a small-scale operation, there are numerous resources to manage, from workers to equipment and ventilation, each of them with their own limitations. Then, in order to keep productivity high, many work sites must be active at any given time, which multiplies the possibilities of resource allocations. Then, the transportation of these resources in the limited space available underground creates even more limitations and complexity, particularly for mining equipment that has low mobility, such as drills. Finally, since several development tasks need to be done before being able to access the mineralized zones, planners must constantly prepare future work places to always ensure that mineral resources are accessible. For all of these reasons, developing precise short-term planning can quickly become a difficult task that uses up a lot of time of qualified planners. Thus, planners have very little time for optimizing short-term plans and often use the first scenario found to achieve medium-term objectives over the time period.

In the following sections, the data set used to test our model will be described, followed by a review of the current literature on the subject of underground mine planning. The model with its possible extensions and modifications will then be presented with the computational results and comparison of its different applications. A brief discussion of the outcome and of future work will then conclude the article. First, a short description of some of the terms and concepts used in the article will be provided.

4.2.1 Terms and Concepts

The mining industry is a very singular one with many terms and concepts being used in no other domain. This section will introduce the reader with the basic notions needed to understand the rest of the work presented in this article.

Three time frames are generally used in the mining industry when it comes to planning activities. The first, strategic or long-term planning, is used to describe objectives over periods of more than a year. This is a global estimation of the operations over the course of a mine's life. Then, the first periods of long-term planning are split into tactical or medium-term planning. At this level, objectives and targets become more precise, but are still estimates. Typical periods for medium-term planning are generally between one and three months. Finally, medium-term planning is separated into short-term or operational planning in periods ranging from hours to a month. At this level of planning, resources are dispatched and there is maximum precision.

As for the type of work, activities in an underground mine are typically separated into two categories: development and production. Development corresponds to all of the excavations done in rocks that have no economic value, called waste. Developments are necessary expanses to efficiently reach and extract the rocks with economic value, called ore. Excavations in this type of rock are called production or stopes. Development and production are carried out using different equipment and techniques. The former usually aims at minimizing rock excavation for a given horizontal or vertical advance and the latter generally aims at maximizing the ore extracted from each blast. Developments also are normally more supported and secured than their production counterparts due to the fact that workers use these openings to perform different tasks, and thus, are exposed to risk. In order to keep the mill and processing plant active, a minimum amount of production has to be done in each time period. This minimum value is often referred to as mill feed objective.

A vein is a relatively narrow ore body extended over a plane in the bedrock, which is very common for precious metal deposits. This type of body and the properties of gold mineral are a unique problem for various reasons. First, contrary to operations dictated by demand in minerals (e.g. iron, copper, coal), gold operations have a virtually limitless demand. No matter how much ore is extracted, the mineral content will be sold at the market value. In terms of planning, this means that the objective is to extract as much ore as possible rather than to fit the demand. This also makes the planning of development much more critical, since accessible ore zones are extracted as quickly as possible, development is never ahead of production and new zones must be constantly developed in order to sustain future production.

4.2.2 Dataset

In order to test the model, a mine plan was developed based on data from an operating Canadian gold mine. A list of operations and equipment were created to fit as closely as possible with real-world values. Here, we will provide readers with a short description of the activities to be performed at the mine that are relevant to short-term planning.

Mine Layout

The project starts with the excavation of the main shaft. Once the depth of the ore body is reached, stations are excavated horizontally as links between the levels and the shaft. Then, drifts are developed between the stations and the ore body. Permanent openings such as garages and refuges are disposed along these drifts. Ventilation shafts are dug from the drifts to the surface to allow fresh air intake from the surface. Once the ore zones are reached, ramps are made to allow the development of drifts called sublevels at a specified height along the veins. Ore and waste passes are excavated between levels and sublevels to transport ore and waste material from the stopes to the shaft. From the sublevels, ore accesses are prepared depending on the mining method used. Figure 4.1 shows an isometric view of the mine layout with its developments and stopes.

The development part of the mine is made of 275 sites including segments of drifts, ramps, ore and waste passes, ventilation shafts and ore access. As for the production, there are 110 stopes in total. Table 4.1 gives a summary of the quantity and total tonnage of the different types of excavations in the mine.

Mining Methods

Two mining methods are used in the mine, backfilled Long-Hole and Cut-and-Fill. The Long-Hole method starts with the extraction of accesses over and under the section of the ore body that is targeted. Then, holes are drilled from these accesses and filled with explosives by the Long-Hole production crew. After the rock is blasted, the resulting fragmented rock is moved by haulage equipment to the closest ore pass. The backfilling crew then comes into action and fills the hole with cemented backfill. A period of three weeks must then be allowed for the backfill to reach its full strength. Therefore, no activities are allowed in bordering stopes during these three weeks. For global stability reasons, Long-Hole stopes are usually extracted along a pyramidal sequence as shown in Figure 4.2, where each square represents a stope in a vein's sublevel and the number, its order of extraction.

The Cut-and-Fill method starts with the excavation of accesses from the side of the vein. A drift with variable dimensions is then excavated following the vein. When the total length is reached, the drift is filled with cemented backfill, and another drift is excavated over the last one. The full height of the vein is extracted by a series of superposing backfilled drifts.

Figures 4.3 and 4.4 illustrate the relations between the different parts of the mine for the development of Cut-and-Fill and Long-Hole stopes, respectively. For Cut-and-Fill, a drift

Table 4.1 Mine Summary

Site Type	Quantity	Total Tonnage (tons)
Shafts	16	45352
Permanent Openings	7	24490
Drifts	84	96368
Ramps	30	117228
Ore/Waste Pass	35	13051
Ore Access	103	132205
Cut-and-Fill Stopes	39	186328
Long-Hole Stopes	71	216127
Total	385	831154

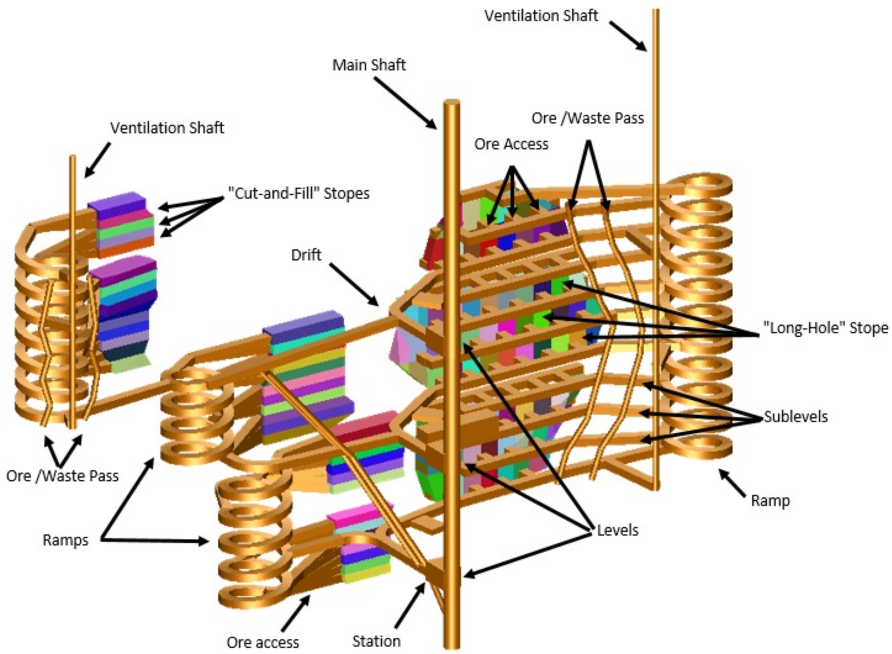

Figure 4.1 Mine Layout

24	23	20	14	9	13	19	21
22	17	12	8	4	6	11	18
16	10	7	3	1	2	5	15

Figure 4.2 Typical Long-Hole Pyramid Mining Sequence

coming from the shaft is dug first, followed by the development blocks for each sublevel as presented in Figure 4.3. A section of ramp and its corresponding ore and waste passes are excavated in order to connect to a small drift near the vein. From this drift, ore accesses are dug to reach each of the sublevel's stopes. For the Long-Hole stopes, a different model of development blocks is used, which is presented in Figure 4.4. For each sub-level, a drift is excavated from the ramp in a direction parallel to the vein length. From this, ore and waste passes to the upper sub-level can be excavated, along with the ore accesses for each of the stopes on this sub-level. In order to start the extraction of a stope, the upper sublevel's ore access from the following development block must also be completed.

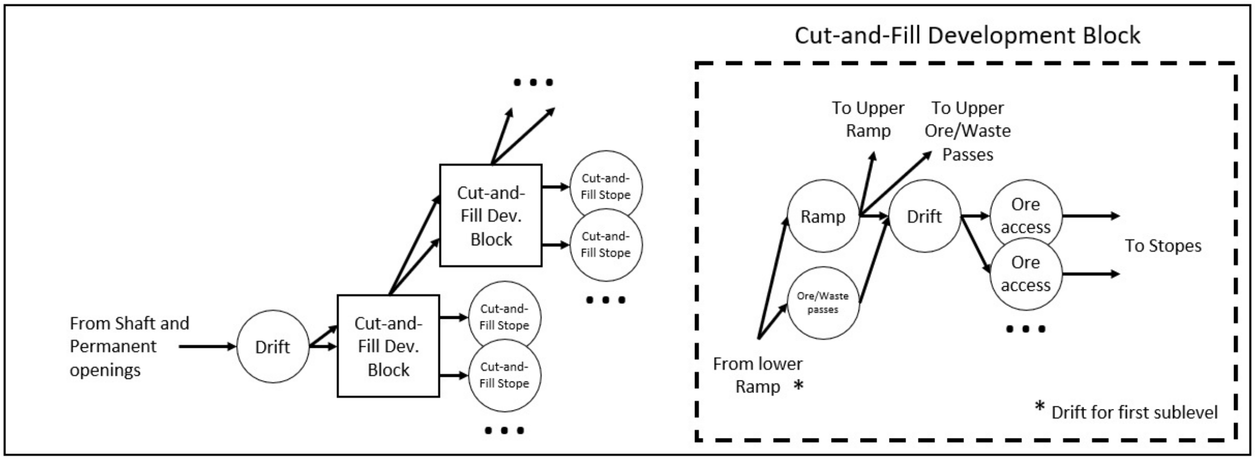

Figure 4.3 Cut-and-Fill Precedencies

Crew

There are six types of crews at work in the mine. What is understood as a “crew type" is either specialized workers or the equipment needed to perform certain operations in the mine. Most developments require more than one blast, meaning that in order to complete the excavation of the sites, all activities must be repeated one after another as many times as the number of blasts. Since blasting happens only twice per day, between shifts, and all activity cycles must finish with a blast, progress in developments is often more limited by the number of blasts than the length of the work to be performed, i.e. the total work time in the site required for one blast is lower than the available time in the shifts. In some sites such as ramps, the experience tells us that a maximum of one blast every two shifts is possible in order to complete all activities, further limiting the progression. For Long-Hole stopes, only one blast is required, thus the main limiting factor is the time needed by the crew to perform their tasks. Table 4.2 provides an overview of the different types of crews working in the mine with their workplaces and the average time spent at each site.

4.3 Literature Review

Planning in the mining industry was traditionally based on the experience of planners and estimations from previous projects. In recent years, however, more and more tools have been developed in order to automate and optimize this process. Since the first publication in the 1960s from [12], considerable progress has been made, driven by developments in operation research and an increase in computational power. For the most part, the application of

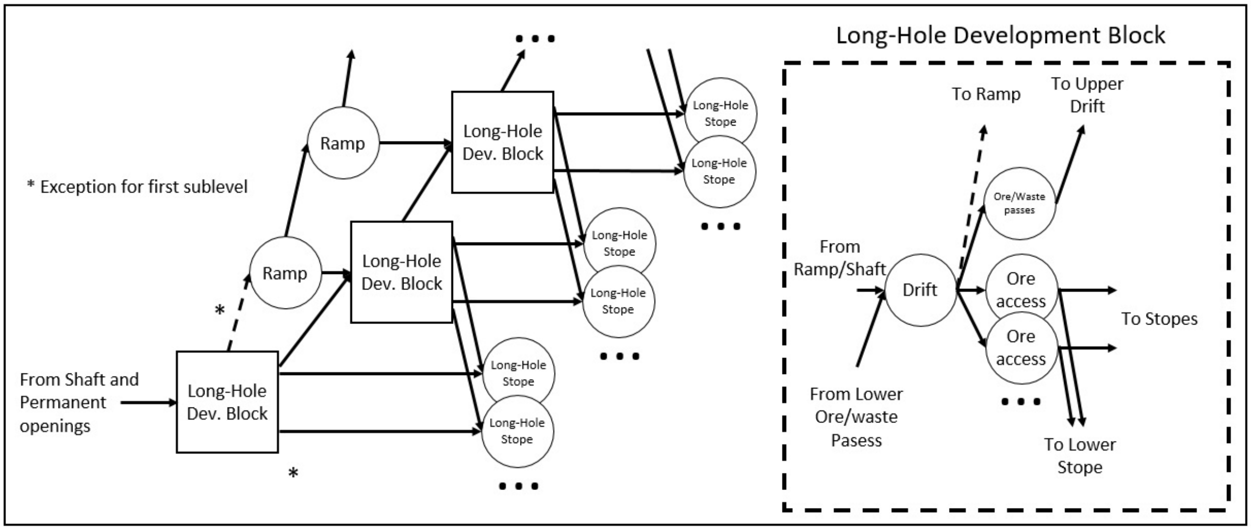

Figure 4.4 Long-Hole Precedencies

optimization in the mining industry concerns open pit projects. One reason for this, as mentioned in [16], is that underground projects are constrained by many more factors than their surface counterparts. Moreover, applications of optimization models to underground mines have to be site specific due to the numerous mining methods and rock haulage systems used in the industry. The most recent literature review on the subject includes [15], which presents a review of optimization in natural resources with a section dedicated to mining, [16], which discusses the advances of optimization in mine planning, and [30], which provides a review specifically on the subject of optimization in underground mines.

4.3.1 Long-Term Planning

From the literature available on underground mine planning, most literature concerns long-term planning. A good example of a long-term application can be found in [71], in which the authors present a model to optimize the starting time in different parts of an underground coal mine. The instances are then solved using a method involving Benders' decomposition to find bounds on the solution and accelerate solving the problem. [33] and [32] then present methodologies to reduce the computational time of general long-term planning models. Similarly, authors in [34] develop a "Greedy randomized adaptive search" procedure to accelerate the solution of a model developed for a copper mine.

Following these, [35] present a classic and improved formulation of the long-term planning model that assigns the different resources and equipment needed for every site. The classic

Table 4.2 Crew Summary

Crew Type	Workplaces	AVG. Time per Site (hours)
Backfilling	Cut-and-Fill and Long-Hole Stopes	87
Haulage	All sites except Shafts	63
Long-Hole Production	Long-Hole Stopes	501
Raise	Ore/Waste Passes	125
Horizontal Drilling	Permanent Openings, Drifts, Ramps, Ore Access and Cut-and-Fill Stopes	104
Shaft	Shafts	126

model simply assigns a binary variable to every activity to be done in every stope and the variation uses a single binary variable for all activities under a more restrictive hypothesis. On a larger scale, [36] gives a model of optimization with low resolution for a large mining complex, including many open pit sections as well as underground parts. A year later, [38] creates another model of optimization for a mining complex with open pits and underground parts that maximize net present value with a variable cut off grade. In a subsequent article, [39] modify the model from [38] to consider geological uncertainty. [46] show the value of optimizing the shape of stopes in parallel with planning by providing a model that optimizes both with results that prove an increase in value. [72] then propose a unified formulation for the long-term planning problem with a simpler notation for resources and a modified version that provides optimization, while respecting a block selection within the mine.

These articles show that previous work mostly considers low resolution mining units, e.g. sections of a mine project instead of work places, with constraints on global limitations. The general trend in more recent works is now to either implement resource-specific constraints like in [35] and [72] or to consider bigger problems involving multiple mines and processes as in [36] and [38].

4.3.2 Medium-Term Planning

Less work on medium-term planning is available than for long-term planning. Nevertheless, [73] present a model specific to the Stillwater mine for time units of three months. The model is then used to evaluate different investment scenarios by extending planning horizon, changing variables and modifying parameters. Then [43] and [44] give two adaptations of the long-term planning model developed for the Kiruna mine in Sweden. Both models' decision variables represent the option of whether or not to start the extraction of ore in all possible extraction points although [44] allow for more precision. [43] then use aggregation and a heuristic to solve the model and [44] use acceleration techniques to reach optimality in reasonable time. Some years later, [45] explained an even more effective heuristic to solve the model by [43] which is based on multiple solves of the problem with different parts of the objective function.

With shorter periods, these models are all more precise than the ones for long-term planning; however, the cost of this level of precision is that either heuristics have to be used to solve the models like in [45] or that the area of application must be limited to certain parts of the mine like in [44] where optimization is focused on the production.

4.3.3 Short-Term Planning

Very limited work exists in short-term underground mine planning. Some literature focuses on real-time optimization as reviewed in [52], but these problems are more about equipment fleet dispatch and are very different than scheduling problems like ours. Nevertheless, [49] present an integrated short- and medium-term optimization model for production. Decision variables on start time for stope developments and excavations are used to smooth the variations on the mill feed and maximize net present value (i.e. the sum of discounted revenues and expanses). It is tested on a conceptual 30 stopes model resulting in a small increase in net present value and fewer mill feed variations compared to separate short- and medium-term planning. Another model can be found in [51] with its application to short-term scheduling at the Lisheen mine in Ireland. Decision variables also dictate the starting time for the excavation of each part of the production. A heuristic is then used to solve the problem.

In all of these models, from long- to short-term, the following hypothesis is posed: once an activity begins at a location in the mine, it is executed for a fixed duration until it is over. This can be a good estimate when activity durations compared to period length are small, as in long-term planning. It can also be applied when the emphasis is put on production where activity durations are similar as in the short-term models cited, but in some cases, as the one presented in this paper, it is problematic. The main reason for this is that when considering development, activity length can vary greatly, creating gaps between activity ends and starts. Since most development equipment is required to visit more then one site in a single shift to reach maximum productivity, preemption does not create any additional setup or moving time, and is more representative of a real mine operation.

The necessity for preemption in underground mine planning can be displayed with a simple example. Figure 4.5 shows a simplified precedence network for the development of a long-hole vein (where some of the development has already been completed) with the duration of each activity. In this example, Sub-Level 1.1 is already accessible as well as Ramp 2 leading to Sub-Level 2.1. Sub-Level 1.1 and Sub-Level 2.1 must be completed in order to be able to start excavating Ore Access 1.1 and Ore Access 2.1. The rest of the Sub-Levels (Sub-Level 1.2 and Sub-Level 2.2) must then be completed before production can start in the vein. The entrance of Drift 1 leading to another vein is also accessible at the beginning of the example.

The horizontal drilling equipment currently used to excavate these sites can complete the drilling in 1.5 sites per work shift, with each site having a maximum rate of advance of one blast every two work shifts. This means that in order to reach full productivity, each drill must have three available sites to work at i.e. three sites will be blasted every day. Considering this limitation, Figure 4.6 gives a Gantt chart showing an optimal planning of all activities in order to start production at the vein as early as possible.

In this example, preemption is used in two sites for different reasons. In Drift 1, production starts during the first week even if it will be completed five weeks later in order to keep the drilling equipment at full productivity while no other sites are available. The other site where preemption is used is Sub-Level 1.2. In this case, preemption is necessary to prioritize sites that need to start as early as possible in order not to delay the start of production. Hence, at the end of week 3 when Sub-Level 2.2 and Ore Acces 2.1 are made available, they are given the priority, pausing the activities in Sub-Level 1.2 for almost two weeks. In this example, where preemption is allowed, production in the vein could start as early as the seventh week where as if it was not, it would be delayed by 1.25 weeks.

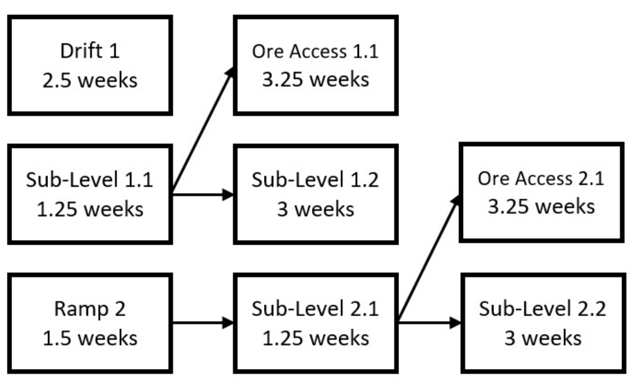

Figure 4.5 Preemption Exemple's Network

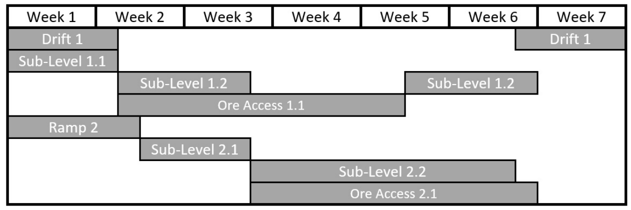

Figure 4.6 Preemption Exemple's Gantt Chart

4.4 Model

The model presented in this article addresses this problem specifically by using a mix of integer and continuous variables to create the activity schedule. It uses one-week periods to create a feasible short-term schedule based on medium-term objectives. The choice of period length was based on planning practices used in many mines where planners create schedule periods of one to two weeks. Each location where extraction activities are planned is called a site and given an index number (s) and each week over the planning period is given an index (t). Veins (v), levels (l) and crew types (c) are also given an index number. The model makes a distinction between extraction, haulage and backfilling activities using continuous variables to represent the progress of each one of them. Haulage activities are performed by haulage equipment, backfilling by backfilling crews and extraction by one of the other four crew types, depending on the type of site. In the definitions below, extractions, haulage and backfilling activities are abbreviated by E, H and B respectively. For sites where many small blasts are required, e.g. drifts and ramps, the haulage variable is made equal to the extraction variable since the progress of extraction and haulage activities are tied together. For sites requiring a single big blast, e.g. Long-Hole stopes, extraction and haulage are independent since the two activities must be completed one after another.

An important parameter is T_s^α which represents the shortest possible span between the beginning and the completion of an activity α in a site s. As mentioned before, the progress rate in each site can be limited by either the number of blasts or the working time, so this parameter corresponds to the maximum between these two limits. As an example, consider a ramp needing 110 hours of horizontal drilling and 17 blasts to complete. Knowing that a blast can only happen once every two shifts in ramps and that a work shift is 10 hours long, T_s^α will be equal to 340 being more limited by the blasts than the total working time. Each site has an objective parameter for each time period, corresponding to the total tonnage of the site discounted over time. The discounting helps to prioritize solutions that finish tasks earlier. A list of sets, parameters, variables and constraints used in the model follows with their definition.

4.4.1 Sets

$\mathcal{E}^S$: Set of all sites
$\mathcal{E}^T$: Set of all time periods
$\mathcal{E}^V$: Set of all veins
$\mathcal{E}^L$: Set of all levels
$\mathcal{E}^V_v$: Set of sites located in vein v
$\mathcal{E}^L_l$: Set of sites located in level l
$\mathcal{E}^H$: Set of sites where rock haulage is separate from the extraction
$\mathcal{E}^B$: Set of sites where backfilling is required
$\mathcal{E}^{NoInt}$: Set of sites to be mined without interruptions
$\mathcal{E}^O$: Set of sites containing ore
$\mathcal{P}^P_s$: Set of sites preceding site s
$\mathcal{P}^S_s$: Set of sites succeeding site s
$\mathcal{P}^B_s$: Set of sites where backfill cannot happen within three weeks before the extraction of site s
$\mathcal{P}^{Stope}_s$: Set of sites preceding site s in Long-Hole veins stope order
$\mathcal{G}$: Set of all three activity groups i.e. {E,H,B} where E stands for Extraction, H for Haulage and B for Backfilling.
$\mathcal{S}^\alpha$: Set of crew types in activity group $\alpha \in \mathcal{G}$

4.4.2 Parameters

A_{ct} : Available crews of type c at time t (units)
C^Q_{st} : Objective coefficient for site s at time t. Equals Q_s for the first time period and decreases with time. (tonnes)
C^O : Penalty incurred for each tonne of ore under the mill feed objective (units)
D_s : Expected dilution in site s (%)
T^α_s : Time span of activity $\alpha \in \mathcal{G}$ in site s (hours)
$T^{\alpha Int}_s$: Time span of activity $\alpha \in \mathcal{G}$ in site s in weeks rounded to the highest integer (weeks)
$T^{\alpha Min}_s$: Minimum time of activity $\alpha \in \mathcal{G}$ in site s for one time period when activity α occurs at this site (hours)
T^{Crew}_{sc} : Number of work hours needed from crew type c to process site s (hours)
T^{Tot}_s : Time span in weeks of extraction, haulage and backfilling in site s rounded to

the highest integer (weeks)
T^{Per} : Number of work hours available per time period (hours)
Q_s : Rock tonnage in site s (tonnes)
Q^M : Maximum possible tonnage extraction in the mine for one time period (tonnes)
Q_l^L : Maximum possible tonnage extraction in level l for one time period (tonnes)
Q_v^V : Maximum possible tonnage extraction in vein v for one time period (tonnes)
Q^O : Mill feed objective (tonnes)

4.4.3 Variables

x_{st}^{α} : Fraction of activity $\alpha \in \mathcal{G}$ executed at site s during time period t
χ_{st}^{α} : Binary variable indicating whether or not activity $\alpha \in \mathcal{G}$ is performed at site s during time period t
δ_t : Tonnes of ore not reaching the mill feed objective during time period t

4.4.4 Objective

The objective function is to maximize extracted tonnes as early as possible, while maintaining a minimum amount of ore tonnage to feed the mill. The first part of Equation (4.1) is simply the multiplication of the fraction of haulage completed for each site at each time period by the discounted tonnage for these sites. The point of optimizing discounted tonnage, as one would optimize discounted profit, is to prioritize solutions that get the maximum done as early as possible. The second part of the objective is the multiplication of the variable representing the difference for each time period between the tonnes of ore extracted in the solution and the mill feed objective by a penalty.

$$Max \qquad \sum_{s=1}^{S}\sum_{t=1}^{T} C_{st}^{Q} x_{st}^{H} - \sum_{t=1}^{T} C^{O}\delta_t \tag{4.1}$$

The choice of such an objective over one that would be related to money comes from discussions with mine planners. The first reason is that at this level of planning, i.e. less than six months ahead, most of the economically influential decisions have been made. The amount of equipment and resources available is already fixed, mining methods are decided and the mine layout should not change. Thus, there are very few possible variations in planning that could affect the revenue of the operations. The challenge then comes from ordering the activities so that the production and development objectives are met or exceeded for the available time. The second reason is that a monetary objective would prioritize production over development on a short-term horizon, since production brings large revenue and development costs. Such solutions could produce delays in the development of future ore zones, creating gaps in production on a medium- to long-term time horizon. Monetary objectives are usually considered when planning for the whole life of mine.

4.4.5 Constraints

$$\sum_{t\in\mathcal{E}^T} x_{st}^{\alpha} \leq 1 \qquad \forall s\in\mathcal{E}^S, \alpha\in\mathcal{G} \tag{4.2}$$

$$x_{st}^{E} - x_{st}^{H} = 0 \qquad \forall s\in\mathcal{E}^S \setminus \mathcal{E}^H \tag{4.3}$$

$$x_{st}^{B} = 0 \qquad \forall s\in\mathcal{E}^S \setminus \mathcal{E}^B \tag{4.4}$$

$$\sum_{s\in\mathcal{E}^S} D_s Q_s x_{st}^{H} \leq T^M \qquad \forall t\in\mathcal{E}^T \tag{4.5}$$

$$\sum_{s\in\mathcal{E}_l^L} D_s Q_s x_{st}^{H} \leq T_l^L \qquad \forall l\in\mathcal{E}^L, t\in\mathcal{E}^T \tag{4.6}$$

$$\sum_{s\in\mathcal{E}_v^V} D_s Q_s x_{st}^{H} \leq T_v^V \qquad \forall v\in\mathcal{E}^V, t\in\mathcal{E}^T \tag{4.7}$$

$$T_s^{\alpha} x_{st}^{\alpha} - T^{Per} \chi_{st}^{\alpha} \leq 0 \qquad \forall s\in\mathcal{E}^S, t\in\mathcal{E}^T, \alpha\in\mathcal{G} \tag{4.8}$$

$$T_s^{\alpha} x_{st}^{\alpha} - T_s^{\alpha Min} \chi_{st}^{\alpha} \geq 0 \qquad \forall s\in\mathcal{E}^S, t\in\mathcal{E}^T, \alpha\in\mathcal{G} \tag{4.9}$$

$$\sum_{s\in\mathcal{E}^S} T_{sc}^{Crew} x_{st}^{\alpha} \leq T^{Per}\ A_{ct} \qquad \forall t\in\mathcal{E}^T, \alpha\in\mathcal{G}, c\in\mathcal{S}^{\alpha} \tag{4.10}$$

$$\sum_{\alpha\in\mathcal{G}} T_{s'}^{\alpha} x_{s't}^{\alpha} + T_s^{\alpha} x_{st}^{\alpha} + T_{s''}^{\alpha} x_{s''t}^{\alpha} \leq T^{Per} \qquad \forall s\in\mathcal{E}^S, t\in\mathcal{E}^T, \; s'\in\mathcal{P}_s^P, s''\in\mathcal{P}_s^S \tag{4.11}$$

$$\sum_{j=1}^{t} x_{sj}^{E} - \chi_{st}^{H} \geq 0 \qquad \forall s\in\mathcal{E}^H, t\in\mathcal{E}^T \tag{4.12}$$

$$\sum_{j=1}^{t} x_{sj}^{H} - \chi_{st}^{B} \geq 0 \qquad \forall s\in\mathcal{E}^B, t\in\mathcal{E}^T \tag{4.13}$$

$$\sum_{j=1}^{t} x_{s'j}^{H} - \chi_{st}^{E} \geq 0 \qquad \forall s\in\mathcal{E}^S, t\in\mathcal{E}^T, s'\in\mathcal{P}_s^P \tag{4.14}$$

$$\sum_{j=1}^{t} \chi_{s'j}^{E} - \chi_{st}^{E} \geq 0 \qquad \forall s\in\mathcal{E}^S, t\in\mathcal{E}^T, s'\in\mathcal{P}_s^{Stope} \tag{4.15}$$

$$\chi^E_{st} - \sum_{j=t}^{t+T_s^{Tot}} x^B_{sj} \le 0 \qquad \forall s \in \mathcal{E}^{NoInt}, t \in \mathcal{E}^T \tag{4.16}$$

$$\sum_{t=1}^{T} \chi^\alpha_{st} \le T_s^{\alpha Int} \qquad \forall s \in \mathcal{E}^{NoInt}, \alpha \in \mathcal{G} \tag{4.17}$$

$$\chi^E_{st} + \chi^E_{s't} \le 1 \qquad \forall s \in \mathcal{E}^S, t \in \mathcal{E}^T, s' \in \mathcal{P}^B_s \tag{4.18}$$

$$\chi^E_{st} + \chi^B_{s'j} \le 1 \qquad \forall s \in \mathcal{E}^S, t \in \mathcal{E}^T, s' \in \mathcal{P}^B_s, j \in [t-3, t] \tag{4.19}$$

$$\sum_{s \in \mathcal{E}^O} Q_s x^H_{st} + \delta_t \ge Q^O \qquad \forall t \in \mathcal{E}^T \tag{4.20}$$

$$x^\alpha_{st} \ge 0 \qquad \forall s \in \mathcal{E}^S, t \in \mathcal{E}^T, \alpha \in \mathcal{G} \tag{4.21}$$

$$\delta_t \ge 0 \qquad \forall t \in \mathcal{E}^T \tag{4.22}$$

$$\chi^\alpha_{st} \in \{0,1\} \qquad \forall s \in \mathcal{E}^S, t \in \mathcal{E}^T, \alpha \in \mathcal{G} \tag{4.23}$$

Constraint (4.2) makes sure all sites are excavated, hauled and backfilled at most once. Then Constraints (4.3) and (4.4) fix unused variables for sites where haulage is part of the excavation and where no backfilling is required, respectively. Constraints (4.5), (4.6), and (4.7) limit the amount of ore extracted in the mine, on every level and in every vein. Tonnage limits for the mine are usually dictated by the haulage capacity of the shaft. Limits on levels can be caused by ventilation or limited work space and limits on veins can be from the haulage or capacity of passes. Constraint (4.8) limits the hours of work for excavation, haulage and backfilling to the available hours in a shift while linking the continuous variables x^α_{st} to their respective binary counterparts χ^α_{st}. Constraint (4.9) gives a lower bound for the time planned in a given week. In our case, the lower bounds T^α_s were fixed to the equivalent of one work shift. Constraint (4.10) limits the activities of each crew type according to the number of teams available. Constraint (4.11) limits the total of all activities in a site, its predecessor and its successor to the available time in a period. This constraint is necessary since work at a predecessor and its successor is allowed in a single period, as long as the predecessor is complete at the end of the period. The effect of this constraint is to make sure that work at the successor is done after the excavation of the predecessor and not simultaneously.

Constraints (4.12) and (4.13) impose the right execution order among activities in sites where haulage is separate from excavation and where backfilling is required. Constraint (4.14) makes sure that all predecessors are hauled before the excavation of a site can start and Constraint (4.15) imposes the stope order in each vein by allowing the start of a stope only if its stope predecessor is also started. In most cases, activities in sites requiring backfilling are planned together with very few interruptions in between them for rock stability reasons.

Constraints (4.16) and (4.17) make sure that no notable interruption in activity flow happens in such sites. One might notice that this set of constraints can allow up to a week-long break between all activities in the worst cases, but it is still representative of the mining reality where this type of delay before backfilling is common. Constraint (4.18) verifies that two bordering stopes cannot be excavated at the same time and Constraint (4.19) verifies that at least three weeks, the minimum time for the backfill to solidify, separates the backfilling of a stope and the extraction of a bordering one. Constraint (4.20) fixes the value of δ_t to the difference between the minimum production and the actual production for each time period.

Following discussions with metallurgists from the gold mining industry, it was decided not to include grade control constraints. These constraints, often seen in open-pit mine planning models, give upper and lower bounds on the average grade of material extracted for each period of time. The reason for these constraints is that a steady ore input grade eases the processing of minerals. However for underground gold mines, the balance between development and production often makes it impossible to respect such constraints, due to the relatively small number of accessible stopes at any given time and the high variability of their grades. If such constraints are found to be necessary for a precise application of the model, they could be easily included with constraints of the general form:

$$\sum_{s \in \mathcal{E}^O} (G_s x_{st}^H - L^{up} x_{st}^H) \leq 0 \qquad \forall t \in \mathcal{E}^T \tag{4.24}$$

$$\sum_{s \in \mathcal{E}^O} (G_s x_{st}^H - L^{low} x_{st}^H) \geq 0 \qquad \forall t \in \mathcal{E}^T \tag{4.25}$$

Where:

G_s : Ore grade at site s

L^{up} : Upper bound on average grade for each period

L^{low} : Lower bound on average grade for each period

4.4.6 Planning Constraints Extensions

With the set of constraints presented above, the model is free to develop any part of the mine. However, in the case of realistic applications, a mine planner would use inputs to make the short-term plan fit longer-term targets, address last minute changes in planning or unexpected events. Small changes can be made to the base model to allow it to rapidly find opportunities in short-term planning and test the impact of different scenarios and alternatives.

Three parameters can be added to the model to allow users to impose planning decisions into the solution.

U_s^{ES} : User defined earliest start for site s (week)

U_s^{LS} : User defined latest start for site s (week)

U_s^{LF} : User defined latest finish for site s (week)

Constraints (4.26), (4.27) and (4.28) can also be added to enforce earliest start, latest start and latest finish for each activity.

$$x_{st}^E = 0 \qquad \forall s \in \mathcal{E}^S, t \in \mathcal{E}^T | t < U_s^{ES} \tag{4.26}$$

$$\sum_{j=1}^{U_s^{LS}} \chi_{sj}^E \geq 1 \qquad \forall s \in \mathcal{E}^S \tag{4.27}$$

$$\sum_{j=1}^{U_s^{LF}} x_{sj}^E = 1 \qquad \forall s \in \mathcal{E}^S \tag{4.28}$$

4.4.7 Non-Preemptive Modifications

As mentioned in Section 4.3.3, all of the models available in the literature for short-term planning consider only non-preemptive solutions. Since these models produce optimal solutions in reasonable computational time for underground planning datasets, the interest of developing a new model for our specific dataset could be questioned. In theory, a preemptive model produces better or equivalent solutions but in practice, optimal solutions could be very similar, leading to equivalent solutions. To demonstrate the positive impact of our preemptive model on the solutions found, a new model was developed. This model, based on the one presented earlier in Section 4.4, only allows activities to be performed at the maximum rate and without interruption. The following is a list of the modifications made to the original model, including only the modified parts, with everything else identical.

New parameters were added to the model. F_{sj}^{α} are made of one vector for each site of length $T_s^{\alpha Int}$ with each entry representing a fraction of the total work executed during the j^{th} time period after the beginning of the activity. For example, if a site s takes 350 hours to excavate and there are 140 hours of available work time in each time period, then $T_s^{EInt} = 3$ and $F_{sj}^E = [0.4, 0.4, 0.2]$ for this site.

F_{sj}^{α} : Fraction of activity $\alpha \in \mathcal{G}$ done at site s during the j^{th} time period after it began.

Variables x_{st}^{α} were changed to binary variables representing the start of activity $\alpha \in \mathcal{G}$ at each site.

x_{st}^{α} : Binary variable representing whether or not activity $\alpha \in \mathcal{G}$ at site s starts during time period t

The model's objective was changed to include the new parameters F^H_{sj} to represent the portion of each site hauled.

$$Max \qquad \sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{j=t}^{T} C^Q_{st} F^H_{sj} x^H_{st} - \sum_{t=1}^{T} C^O \delta_t \tag{4.29}$$

Constraints (4.2), (4.3) and (4.4) were included unmodified from the preemptive model. Constraints (4.8), (4.9), (4.17) and (4.18) were not included since the binary nature of the variables and the new parameters were rendering them useless. Constraints (4.5), (4.6), (4.7), (4.10), (4.11) and (4.20) were modified in a way to include the new parameters. For each of them, the variables x^α_{st} and χ^α_{st} from the original model were replaced with:

$$\begin{array}{llll}
Original & & Non-Preemptive & \\
x^\alpha_{st} & \rightarrow & \displaystyle\sum_{j=t-T^{\alpha Int}_s+1}^{t} F^\alpha_{s(t-j)} x^\alpha_{sj} & \forall \quad \alpha \in \mathcal{G} \\
\chi^\alpha_{st} & \rightarrow & x^\alpha_{st} & \forall \quad \alpha \in \mathcal{G}
\end{array}$$

Constraints (4.12), (4.13), (4.14), (4.15), (4.16), (4.18) and (4.19) were replaced with the Constraints (4.30), (4.31), (4.32), (4.33), (4.34), (4.35) and (4.36) as presented below:

$$\sum_{j=1}^{t-T^{EInt}_s+1} x^E_{sj} - x^H_{st} \geq 0 \qquad \forall s \in \mathcal{E}^H, t \in \mathcal{E}^T \tag{4.30}$$

$$\sum_{j=1}^{t-T^{HInt}_s+1} x^H_{sj} - x^B_{st} \geq 0 \qquad \forall s \in \mathcal{E}^B, t \in \mathcal{E}^T \tag{4.31}$$

$$\sum_{j=1}^{t-T^{HInt}_{s'}+1} x^H_{s'j} - x^E_{st} \geq 0 \qquad \forall s \in \mathcal{E}^S, t \in \mathcal{E}^T, s' \in \mathcal{P}^P_s \tag{4.32}$$

$$\sum_{j=1}^{t-T^{EInt}_{s'}+1} x^E_{s'j} - x^E_{st} \geq 0 \qquad \forall s \in \mathcal{E}^S, t \in \mathcal{E}^T, s' \in \mathcal{P}^{Stope}_s \tag{4.33}$$

$$x^E_{st} - \sum_{j=t}^{t+T^{Tot}_s-T^{BInt}_s+1} x^B_{sj} \leq 0 \qquad \forall s \in \mathcal{E}^{NoInt}, t \in \mathcal{E}^T \tag{4.34}$$

$$x^E_{st} + \sum_{j=t}^{t+T^{EInt}_s-1} x^E_{s'j} \leq 1 \qquad \forall s \in \mathcal{E}^S, t \in \mathcal{E}^T, s' \in \mathcal{P}^B_s \tag{4.35}$$

$$x^E_{st} + x^B_{s'j} \leq 1 \qquad \forall s \in \mathcal{E}^S, t \in \mathcal{E}^T, s' \in \mathcal{P}^B_s, j \in [t-2-T^{BInt}_s, t] \tag{4.36}$$

Finally, Constraints (4.21) and (4.23) were replaced by:

$$x_{st}^{\alpha} \in \{0,1\} \qquad \forall \quad s,t,\alpha \in \mathcal{G} \tag{4.37}$$

4.5 Computational Study

The following section will present the results of the application from different versions of our model. All results presented in this section were performed with a computation time limit set to 1800 seconds. A solution less than 0.25% from the upper bound was also considered optimal. This choice of optimality gap was based on the fact that, as will be seen in later analysis, solutions rarely improve past this point and on average, for solution values presented, 0.25% represents less then one tenth of the tonnage of an average long-hole stope. These tests were performed on a computer with an Intel(R) Core(TM) i7-4770 CPU @ 3.40GHz and 16 GB of RAM , using the branch-and-cut algorithm from IBM ILOG CPLEX Optimization Studio version 12.7.1.0 with up to 8 threads. Results of the base or unmodified model will be presented first with a series of tests including different starting scenarios and random variations. A non-preemptive version will then be compared to the preemptive one, and finally, a case study will be shown with tests demonstrating its application possibilities.

4.5.1 Results

Three scenarios were created to simulate the different states in a mine life. The scenarios were made by varying the progress of operations in the mine. The first scenario is the equivalent of starting the operation from scratch, where all the sites are intact and a lot of development has to be done before reaching the ore zones. The second scenario simulates a more advanced state where 66 sites have already been excavated. Most of the main developments are completed but there are still local developments to excavate before production can start. The third scenario, where 208 sites are considered done, represents a state where production has already started and little development is left to do.

For each of these scenarios, the planning horizon was gradually increased from 12 to 24 weeks. Two mines generated from random variations of the original were also tested for each scenario and time period to get more sampling data. Case 1 represents the basic mine model presented above while Cases 2 and 3 were produced by randomly increasing or decreasing the tonnage of each site. Table 4.3 displays the total number of variables, number of binary variables and number of constraints for each combination of scenario and planning horizon for Case 1. Note that an early-start was computed for each site at the beginning of each test in order to reduce the number of variables included in the model. Using this technique the number of variables entered in the model was reduced by up to three times. The application of this preprocessing had very little effect on the computation times though, since the commercial solver does automatically apply preprocessing techniques that result in similar variable number reductions. The values presented in Table 4.3 were obtained after the application of early-starts. These numbers are very similar for all cases.

Table 4.3 Formulations Properties of Case 1

Week	Variables	Binaries	Constraints
Scenario 1			
12	7818	3903	53656
18	11920	5951	79816
24	16286	8131	105976
Scenario 2			
12	9616	4802	48982
18	16964	8473	72904
24	25246	12611	96826
Scenario 3			
12	7598	3793	31648
18	12406	6194	47116
24	17568	8772	62584

Table 4.4 displays the computational results of the application of our model to the different datasets. The first three columns represent the characteristics of the different datasets, where Case represents the case number, Sce. the scenario number and Week the number of weeks in the planning horizon. The Obj and % display the final objective value of the solution found and the remaining relative gap between the objective value and the best upper bound available in the branch-and-cut algorithm. Columns LP Time and LP Gap show computational time and the integrality gap of the linear programming relaxation for the problem. Node Pro. and Node Rem. represent the number of nodes processed and remaining in the branch-and-cut tree. Finally, $Time_{5\%}$, $Time_{1\%}$ and $Time_{0.1\%}$ represent the time needed to find a solution respectively within 5, 1 and 0.1 % of the final solution while $Time_{Sol}$ and $Time_{Tot}$ show the time needed to find the final solution and the total computational time. The time to reach the final solution is sometime smaller than the total time since even when the optimal solution is found, the branch-and-cut algorithm usually needs more time to lower its upper bound and prove the optimality of the solution.

Table 4.4 Computationnal Results

Case	Sce.	Week	Obj	%	LP Time	LP Gap	Node Pro.	Node Rem.	$Time_{5\%}$	$Time_{1\%}$	$Time_{0.1\%}$	$Time_{Sol}$	$Time_{Tot}$
1	1	12	18825	0.00	0.00	0.73	0	0	0.02	0.02	0.02	0.02	0.02
1	1	18	24481	0.00	0.00	0.80	0	0	0.04	0.04	0.04	0.04	0.04
1	1	24	53026	0.00	0.00	3.39	0	0	0.07	0.07	0.07	0.07	0.07
1	2	12	60219	0.12	0.01	13.33	0	1	0.19	0.19	0.19	0.19	0.19
1	2	18	95024	0.25	0.02	12.33	13	4	2.77	2.77	2.77	2.77	2.86
1	2	24	132873	0.25	0.06	13.24	2740	1270	6.24	6.24	12.34	22.44	24.41
1	3	12	79830	0.25	0.02	7.45	3709	79	1.04	4.99	4.99	8.31	10.25
1	3	18	118011	0.21	0.09	10.43	1913	753	7.54	14.94	15.27	19.44	20.11
1	3	24	158252	0.30	0.21	14.61	22680	9773	41.90	52.61	88.53	837.28	1800
2	1	12	18700	0.09	0.00	1.12	0	1	0.03	0.03	0.03	0.03	0.03
2	1	18	24221	0.00	0.00	1.46	0	0	0.08	0.08	0.08	0.08	0.08
2	1	24	48355	0.01	0.00	7.27	0	1	0.21	0.21	0.21	0.21	0.21
2	2	12	82615	0.25	0.01	22.61	1399	864	0.92	0.92	0.92	1.40	2.05
2	2	18	124015	0.25	0.04	22.98	21452	7122	3.23	3.23	5.58	44.59	110.24
2	2	24	158025	0.84	0.09	32.48	66463	23160	5.63	11.20	19.32	1630.56	1800
2	3	12	77953	0.25	0.02	46.43	1238	213	1.49	1.49	1.49	2.42	2.54
2	3	18	107495	0.25	0.05	50.98	7043	116	5.54	5.57	9.10	16.20	38.59
2	3	24	133604	0.25	0.10	49.47	31944	11084	15.40	32.23	129.30	219.34	612.16
3	1	12	18132	0.00	0.00	0.87	0	0	0.03	0.03	0.03	0.03	0.03
3	1	18	25792	0.00	0.00	0.52	0	0	0.04	0.04	0.04	0.04	0.04
3	1	24	35994	0.15	0.00	17.16	0	1	0.07	0.07	0.07	0.07	0.07
3	2	12	79577	0.23	0.01	30.42	9	6	0.59	0.59	0.59	0.86	0.86
3	2	18	113345	0.25	0.04	42.66	48498	5385	3.03	3.03	3.51	112.73	157.00
3	2	24	143372	0.46	0.10	51.52	153084	59336	4.97	9.07	10.18	142.42	1800
3	3	12	81687	0.25	0.02	46.31	14953	6245	1.91	1.91	1.91	5.47	43.34
3	3	18	107860	0.25	0.05	55.28	3609	320	3.65	5.07	5.07	25.96	30.34
3	3	24	123334	0.25	0.06	74.11	52382	13141	5.14	20.78	20.78	399.78	812.68

The first thing that can be observed from these results is that all test with a planning horizon of up to 4 months, or 18 weeks, were solved to optimality within the time limit. For the test involving 24 weeks, some could not be solved, but all reached solutions within 1% of optimality. From Table 4.3, we see an increase in variables from Scenario 1 to 2 and then a decrease from 2 to 3. This variation is linked to the number of possible sites to mine from. In Scenario 1, a lot of development has to be completed in order to reach the veins where most sites are located, eliminating many variables from sites located in these veins in the first time periods. In Scenario 2, most of the sites are accessible for the given time horizon, leaving less possibilities for variables eliminations and in Scenario 3 less sites are left to be mined decreasing the number of variables. For the constraints, we see a decrease from Scenario 1 to 3 since the chains of precedences between sites and crews, the main source of constraints, decreases in length from Scenario 1 to 3.

Interestingly tough, the complexity of the problem is not directly linked to the number of variables or constraints in the problem. The number of variables and constraints does increase with the length of the planning horizon as does the complexity of the problem when considering cases and scenarios one by one. But the scenarios that were the hardest to solve are not necessarily the ones with the most variables. As seen from Table 4.3 and Table 4.4, even if instances of Scenarios 2 and 3 were in general longer to solve than instances of Scenario 1, their number of variables and constraints are lower. This is explained by the fact that in Scenarios 2 and 3, even if many sites are already completed, the decisions to make are much more interlinked and planning possibilities are greater.

Looking at the LP Time and LP Gap column from Table 4.4, it can be noted that computational times for the linear relaxations are all under 0.21 seconds, which demonstrate that the relaxations are relatively simple to solve. When looking at the gap of these relaxations, the difference between the optimal solution value of the relaxations and the integer solutions are in some cases extremely small; 0,52% for Case 3, Scenario 1 with 18 weeks, and in other very large; 74% for Case 3, Scenario 3 with 24 weeks. The explanation for this comes from the precedence structure of the problems and the efficiency of the early-starts strategy to eliminate variables. Since planning possibilities are limited in the first scenario, the early-starts computed for each site are almost identical to their starting time in the final solution and few integrality constraints are violated. On the opposite, when possibilities are multiple and the early-starts computed in preprocessing are much less representative of the final solutions, as in Scenario 2 and 3, the integrality gap increases. When not limited by early-starts, the reason why linear relaxation gives such poor solutions is that the most constraining elements of the problem are the precedences between sites. Since the constraints forcing precedences rely on the integrality of the binary variables, the linear relaxation does not consider one of the major restrictions on the solution. From the analysis of the relaxations, one can see that the solutions are characterized by long series of fractional binaries instead of integers. Figure 4.7 shows results from Case 2, Scenario 2 with 18 weeks, representing χ_{st}^{R} for stope va1_l250_s1_01X obtained by the LP relaxation and by the integer problem. This figure is representative of the effect of the relaxation of the precedences constraints. In the relaxed problem, a site can start as early as the fourth week whereas in the integer problem, it only starts at week 17. This also results in 13 stopes being mined in the relaxed problem where only 4 are mined in the integer problem.

From the columns Node Pro. and Node Rem. of Table 4.4, it can be observed that some instances did not need the branch-and-cut algorithm to get to the optimal solution. This is due to the fact that before starting the algorithm, CPLEX generates many cuts and applies different pre-processing techniques to reduce the problem size and generate a good starting solution if possible. In these cases, due to the limited number of planning possibilities, these good starting solutions happened to be optimal.

For more complex scenarios, the number of processed nodes increases rapidly with increasing planning horizons. Two reasons explain this tendency. First, since the relaxation of the problem gives a solution very different from the final solution, the branching tree has to be explored deeper in order to find integer solutions. Moreover, the integrality gap being larger, the pruning of the branches of the tree is less efficient. Second, the large number of branches is caused by the problem being almost, but not exactly, symmetrical in many points. Some parameters are available in CPLEX to help limit the effect of symmetry in the problem treated. Many configurations of these parameters were tested on instances of our problem but none made a significant difference on the computational time.

From columns $Time_{5\%}$, $Time_{1\%}$, $Time_{0.1\%}$,$Time_{Sol}$ and $Time_{Tot}$, it can also be observed that good solutions are found early in the resolution. For all datasets, a solution 0.1% from optimality can be found in the first three minutes. Then, for all datasets that were solved to optimality, the optimal solution was found in less than 400 seconds with the rest of the time spent on proving optimality. This implies that in the context of an industrial use of the

Weeks	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
LP Relaxation	0	0	0	0.1	0.2	0.3	0.3	0.3	0	0	0	0	0	0	0	0	0	0
Integer Problem	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1

Figure 4.7 Stope Results for χ_{st}^{R} for Relaxed and Integer Models

model, time limits could be set lower and still provide good solutions.

4.5.2 Preemption and Non-Preemption Comparision Results

Case 1, corresponding to the base situation, was used to make a comparison between the preemptive or base model and the modified non-preemptive model. The scenarios and numbers for the time period are the same as the ones used for the model presented in Section 4.4. The results are shown in Table 4.5, where the columns NP and P represent the solutions of the non-preemptive and preemptive models and the % column is the difference in percentage between them.

Table 4.5 Preemptive and Non-Preemptive Model Results

Scenario		**1**			**2**			**3**	
Period	**NP**	**P**	**%**	**NP**	**P**	**%**	**NP**	**P**	**%**
12	17000	18824	10.7	58880	60237	2.3	77130	79829	3.5
18	23820	24481	2.8	90492	95034	5.0	116089	117983	1.6
24	40000	53194	32.5	124832	132874	6.4	154217	158367	2.7

As expected, since it is theoretically impossible for a preemptive solution to be worse than a non-preemptive, the results of the non-preemptive model are all lower than the preemptive ones. Even though some results are fairly close, it still demonstrates that a non-preemptive model could not be used as an estimate in our situation. Results from the first scenario might seem surprising, going from a 10.7% gap to 2.8% and back to 32.5%. This can be explained by the fact that the non-preemptive model creates artificial delays in scheduling. Figure 4.8 provides a good example of this kind of delay, showing the resulting schedule for vein Va1 ore passes in the preemptive and non-preemptive models. This series of ore passes, all similar in size, are precedent to one another. Since it takes less than a week to complete each of them, the preemptive model allows the successors to start in the same period. However, the non-preemptive model does not, since the two cannot be completed entirely in the same week. Ultimately, these delays add up and create a difference of more than two weeks over the whole ore pass chain. What is observed in Scenario 1 is the result of a bottleneck in the development chain around week 18 for the preemptive model, which makes it possible for the non-preemptive model to catch up. In the instance with 24 weeks, the preemptive model is then able to get past this bottle neck while the non-preemptive is not, creating a much bigger gap between the values of the solutions.

Preemptive										
Sites\Week	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
ore_pass_va1_1	1									
ore_pass_va1_2	0.33	0.67								
ore_pass_va1_3		0.6	0.4							
ore_pass_va1_4			1							
ore_pass_va1_5				1						
ore_pass_va1_6				0.33	0.67					
ore_pass_va1_7					0.6	0.4				
ore_pass_va1_8						0.83	0.17			
ore_pass_va1_9							1			
ore_pass_va1_10							0.2	0.8		

Non-Preemptive										
Sites\Week	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
ore_pass_va1_1	1									
ore_pass_va1_2		1								
ore_pass_va1_3			1							
ore_pass_va1_4				1						
ore_pass_va1_5					1					
ore_pass_va1_6						1				
ore_pass_va1_7							1			
ore_pass_va1_8								1		
ore_pass_va1_9									1	
ore_pass_va1_10										1

Figure 4.8 Ore Pass Results for Preemptive and Non-Preemptive Models

4.5.3 Case Study

A set of scenarios was prepared to display how the model would perform under real-world conditions. The scenarios were derived from common operational constraints described by mine planners. Using the base case, Case 1 in Table 4.4 and a starting situation corresponding to Scenario 2 with 18 weeks, Table 4.6 shows the results of four different scenarios. For each scenario, the value of the objective function and the number of active sites are given. The total tonnage extracted is also given in tonnes, and split between ore and waste. Finally, the sum of penalties (i.e. the sum of δ_t) and the computation times are given in the last two columns.

The first scenario, called Origin, is the base scenario that was used in Subsection 4.1. This result produced without input from the planner would be less likely to concord with long-term planning or would be through luck, since the model only produces optimal solutions for the time horizon it considered. Scenario Va1 is a representation of the impact of imposing rapid development of the first three sublevels of vein Va1 in the first ten weeks, so that production from this vein can be higher later in time. It can be observed from Table 4.6 that this priority in development comes with a cost, since Scenario Va1 has a lower solution value. The main reason for this is that when prioritizing development in this vein, development of other veins is delayed and so is production. Production from vein Va1 stays the same since the curing time for the first stope's backfilling goes over the time horizon considered.

If the main priority was to develop the ramp climbing on the second level of vein Va1, in order for this example to proceed with definition drilling of the second level, the Ramp scenario would be the optimal solution. It is clear that this scenario is very similar to the original one, simply assigning resources to a different path without restraining other developments. The last scenario, Va1+Ramp, is an attempt at reaching these last two objectives in the same schedule. The model proves very quickly that they are incompatible. The resources necessary to complete the first objective delays the advance of the ramp in the first periods and makes it impossible to complete the ramp in time.

Table 4.6 Case Study Results

Scenario	Obj	Site	Tonnage	Ore	Waste	Penalty	Time
Origin	95034	65	107970	19580	88390	9000	12.4
Va1	90958	83	107642	12540	95102	11000	17.2
Ramp	95044	64	107981	19580	88401	9000	8.7
Va1+Ramp	Infeasible	-	-	-	-	-	0.3

4.6 Conclusion

The motivation behind developing the model presented in this article was to create a tool that is able to quickly optimize scheduling of short-term activities and test alternatives. The results of the application to different phases of mine development prove that the model can be solved to optimality, or near optimality in more complex cases, in a very short time for instances representative of different medium-term periods. Application-oriented scenarios have demonstrated its ability to get the most out of a short-term schedule by quickly determining the outcome of different planning decisions. A comparison with a non-preemptive model also showed the necessity of modeling the problem in a preemptive way, giving both better and more realistic solutions. Nevertheless, the output of this model still relies heavily on the quality of the medium-term planning and the precision of its input parameters, limiting its possibilities. An extension to simultaneously optimize short- and medium-term or a stochastic implementation would help improve the model possibilities. Further research will be needed to ameliorate these aspects.

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CHAPITRE 5 ARTICLE 2: INTEGRATED OPTIMIZATION OF SHORT- AND MEDIUM-TERM PLANNING IN UNDERGROUND MINE

Louis-Pierre Campeau, Michel Gamache, Rafael Martinelli, (2019), "Integrated optimization of short- and medium-term planning in underground mine", International Journal of Mining, Reclamation and Environment, Soumis.

5.1 Abstract

This article describes a new model aiming at optimizing short- and medium-term underground mine scheduling. The complexity of the problem to solve and the frequency at which planners have to revise these schedules are among the main motivation for developing such a model. In order to address this problem, a Mixed Integer Programming model was developed with a variable time discretization to accurately represent both short- and medium-term operational constraints in a single model. Results of a preliminary model are presented with explanations and in-depth analysis. An improved formulation is also described with it's associated results and benefits. Further testing with scenarios similar to long-term planning show very promising results for the possible application of our modified formulation to existing long-term model.

5.2 Introduction

As for many other sectors, the mining industry has seen many changes in the recent years and many more seems to be coming in a near future. In order to increase profitability, more and more mines include automation in their processes. From automated trucks to stope shape optimizers, a decreasing number of decisions are taken without the use of a computer. Among these, underground mine planning is a sector of activity that still lacks the tools to use the power of automation and optimization. This article aims at addressing this particular problem. In order to introduce the reader with all the necessary notions to the understanding of this article, we will first give a short description of some terms and concepts.

Planning in underground mines is made at four different levels. The first, long-term planning, is typically based on yearly time periods and defines general production objectives and development goals for the whole life of mine. The precision of the planning is very limited, considering that the time periods are very long and that the majority of the production planned is based on geostatistical models obtained from a few distant drilling holes. Stopes designs at this level of planning are often very generic, since the final shape of the vein is still uncertain. This planning is usually revised every year or two or as often as notable changes occur in the geological model. The second level, medium-term planning, is usually performed on time periods of one to three months. This planning is more precise than the long-term from its shorter time periods and better defined veins. This schedule is usually revised every three months or so to adjust previsions according to the actual advances in the mine. Short-term planning is the third level of this planning sequence and has a time horizon of one or two weeks. Short-term schedules are usually revised every week. Finally, the first week of the short-term planning is divided into shifts and revised at the end of every shift accordingly to the events of the day. This last level of planning is referred to as real time planning.

The following article will describe a model of mathematical programming designed to optimize simultaneously short- and medium-term planning. As will be demonstrated further, from the limited literature on the subject of underground mine planning, most articles are focusing on long-term planning. Very few models are available for lower planning levels, and there is a great need for more research to develop working models. Two major reasons make these levels of planning challenging. First, a mine is a work environment with many resources to allocate and many possible workplaces. This creates a large number of possibilities in scheduling and increases the planning problem's difficulty. Second, a typical mine layout involves many different ore zones spread across a large area. To reach these zones, series of developments must be excavated long before the stope's extraction can be started. This characteristic makes it even more difficult to plan and optimize activities in underground mines since development activities always have to be ahead of production activities in order to keep a constant production level.

The reason for the choice of an integrated model is that on the short-term level, decisions must take into account the medium-term objectives for development and production to be realistic. Thus, integrating both levels of planning in a single model guarantee that the short-term planning will take in consideration medium-term objective of development and production, and that the medium-term planning will produce feasible and realistic planning for the short-term. Furthermore, it is well known that by solving a problem in separate parts instead of as a whole, optimality can be lost. The normal mine planning process involves the separation of the whole mine planning in four different levels at the expense of optimality. The integration of two of those levels together could create more planning potential.

The benefits from such a model are plenty. Even if the input of a mine planner would still be needed, it would greatly reduce the time taken to produce both short- and medium-term planning. This is even more emphasized by the fact that these two schedules have to be revised very frequently. These constant revisions also leave little time for the planners to optimize short- or medium-term planning. Therefore, the model would help optimize a part of planning that rarely is optimized, by proposing optimal solutions and leaving more time for the planners to work on each schedule. Finally, mine planning is still mostly based on the planners experience and estimates. Our model can help normalize the process and make it less dependent on the planner.

In order to correctly address the problem of integrated short- and medium-term underground mine planning, the model will have to be precise on the short-term to define weekly objective, but with enough foresight to prepare for the development of distant ore zones. It will have to be solvable in less than an hour so that it can be used multiple time in the course of a planner's work shift. Finally, it will have to be able chose a subset of activities to be completed from the larger set of all of the mine's activities so that planners do not have to enter as input a list of activities to complete, which would reduce the optimization possibilities.

The following sections will introduce the reader to a literature review of the models already developed for underground mine planning with a focus on short- and medium-term. A model addressing all of the points presented above will then be introduced and a presentation of its improvement will follow. Results and comparison of the initial and improved model will then be shown and discussed.

5.3 Literature Review

Contrarily to its open-pit counterpart, optimization in underground mines is a much less documented subject. In its review of the current literature and opportunities in this field, [19] attributes this lack of interest to the greater complexity and specificity of underground mine problems. A theory supported by both [15] and [16] in their respective review of operation research in natural resources and in mine planning. In underground mine planning specifically, most of the mathematical models that have been developed have been designed to solve long-term planning problems. These problems are usually solved on a low resolution, i.e. long time periods and large mining units, to compensate for the considerable size of the problems.

[47] propose a standard and general formulation of the underground mine planning optimization model, summarizing it to precedences, upper and lower bounds on resources consumption and unique completion of each mining unit. A second formulation is then introduced to consider selectivity in the mining process, or the ability to select a variable grade from different

stopes. A modified grade/tonnage curve taking into consideration the mining method is used in to model this selectivity while keeping a low level of resolution. Using a similar general definition of the planning problem, [37] present a method based on Lagrangian relaxation to accelerate the resolution of such problems. In the same year, [36] develop a heuristic to solve the long-term planning model of a group of open-pit and underground mines in Chile sharing multiple processes. The principle of the method is based on a rounding procedure of linear programming (LP) relaxations in order to achieve good solutions.

The idea of integrating multiple aspects of mining through a single formulation can also be found in [38]. Instead of integrating multiple mines, the model proposed integrates the selection of a variable cut-off grade to the optimization problem. [39] explore this approach further by adding geological uncertainty into the model solving it as a stochastic integer programming model. In a similar way, [46] integrate another aspect of mining to the planning problem by proposing a model considering variable stopes design. As for all other integration presented here (e.g. cut-off grade and scheduling, multiple mine site scheduling), results of the application prove that the integration of multiple aspect into a single problem yields a sizable increase in the net present value (NPV) of the solutions.

On a finer discretization level, [35] and [45] both propose models for optimization of underground mine schedules with monthly time periods. [35] present a standard and an improved model for underground mine scheduling that aims at maximizing NPV. The improvement is based on the aggregation of multiple binary variables into one for sequences of tasks that are known to follow one another without interruptions. Both models are tested on a conceptual 50 stopes operation over a period of 4 years with noticeable computational time reduction for the improved model. [45] on the other hand describes a heuristic to solve a known formulation of the underground mine scheduling problem that aims at minimizing the deviation from production targets. The heuristic is based on the successive relaxation of constraints related to the different parts of the objective. The information extracted from these relaxations is then used to fix certain variables before a final solve with all constraints is completed. The necessity of the heuristic is then proved by the comparison to the performances of the usual branch-and-bound on large problems, where the former cannot find optimal solutions.

Table 5.1 and Table 5.2 present the short-term models available from the current literature. The first columns of each table refer to the articles in which the models are presented. In Table 5.1, the “Element” column refers to the main commodities being mined in the operation for which the model was developed. The “Mining Method” column lists the mining methods considered by the model, where LH stands for Long-Hole, CF for Cut-and-Fill and RP for Room-and-Pillar. The “Data Source” column indicates from which source the testing dataset was taken.

In Table 5.2, the “Time Unit” column refers to the base unit for time discretization used by the models and column “Horizon” displays the longest time horizon for which the model was solved. The “Objective” column defines what is considered in the objective function of the models, where DE stands for discounted extraction, TD for target deviations and NPV for Net Present Value. What is understood as discounted extraction is any objective where a time-decreasing function of the tonnage of the site extracted is used. Target deviation includes all objective functions where a penalty is included for not reaching pre-defined targets on tonnage or ore quality and the net present value is the discounted value of all activities taking place in the schedule. The “Resource Constraints” column indicates whether the models have limitations on global or individual resources. That is, if similar crews or machines are considered as one resource with an equivalent capacity for all of them (Global) or if each of them is given its own capacity and assigned to specific tasks (Individual). The advantage of considering each machine or crew as an independent resource is that the resulting planning is directly applicable and does not need further processing in order to allocate each resource to work places. The downside of it being that it requires more variables to represent the same situation. The “Planning Variables” column indicates if the planning variables, i.e. the variables used to express the completion of a task, are binary or continuous. The advantage of using continuous variables to express task completion is that it creates more flexible models, where activities starting point and execution rates are not limited by the time discretization. But these additional possibilities also usually come with higher complexity and a need for larger computational resources.

The first entry from [74] is a model applied to a Canadian gold mine dataset containing 385 workplaces and 6 equipment types. The continuous variables express the completion of different activity groups and a limit of 10 minutes is set for computation time. The second entry from [48], describes a model in which continuous variables are used to indicate how much ore has to be moved from different ore movement locations by specific machines. It is then applied to a conceptual operation of 60 ore movement locations with 8 machines and solved in less than a minute. [49] describes a model with similar concepts in which variables

Table 5.1 Short-term models contexts

Model	Element	Mining Method	Data Source
[74]	Au	LH, CF	Canadian Gold Mine
[48]	Cu	LH	Conceptual
[49]	Cu	LH	Conceptual
[50]	Pb, Zn	LH, CF, RP	Lisheen Mine

Table 5.2 Short-term models characteristics

Model	Time Unit	Horizon (Months)	Objective	Resource Constraints	Planning Variables
[74]	Week	6	DE+TD	Global	Continuous
[48]	Shift	2	TD	Individual	Continuous
[49]	Week+Month	18	NPV+TD	Individual	Con + Bin
[50]	Week	24	DE	Global	Binary

also dictate ore movements performed by specific machines but also includes binary variables to indicate the starts of development activities. The model is solved using two different time periods, week for the first 12 periods and months for the following. It is applied to a conceptual mine with 30 stopes, 21 developments and 5 machines, and solved to optimality. The model presented in [50] uses binary variables to denote the start of mining activities in workplaces. The model was developed for the Lisheen mine approximately two years before its closure, and plans the extraction of the 1193 workplaces left to mine in less than 20 hours of computations. [51] later described a heuristic to improve the tractability of this last model.

On a final note, there are also in the literature some articles covering real time planning in underground mine, like [53], but they will not be covered here since they are a rather different, being closer to job shop scheduling problems.

5.4 Model

From the review of the available models in the literature for short term scheduling, it can be noticed that none of the models are appropriate to solve our problem. The model presented in [74], although very well adapted to our dataset is lacking in foresight to completely cover a long-term period and optimize both short- and medium-term. A medium-term model that does not cover a full long-term period cannot assess the capacity of it's results to achieve long-term objectives.

The models from [48] and [49] being focused on ore movement and production, with little planning of the development would not fit with the reality of our problem. The size of the solvable problem may also be a problem. Finally, the model presented in [50], although working very well for its dataset, does not really fit our problem since the reality of a closing mine where all of the developments are already completed is very different than the reality we are trying to represent here.

Throughout the article, the word "site" will be used to describe any workplace where an activity has to be performed, including stopes. The word "crew" will refer to any team of specialized workers or pieces of equipment. In each site, a specific sequence of crews must be followed in order to carry out all activities linked with this site. This sequence varies accordingly to the nature of the site and one crew must completely finish his activities in order for the next one to start his. The precedence graph $\mathcal{G}_s^{Crew}$ will be used in this article to describe the sequence of activities in a site s where any arc (i, j) indicate that activity i must be done before j. In the same fashion, sites are linked together by precedence relations and all activities from a predecessor site must be completed before any activity of it's successor can be started. The precedence graph $\mathcal{G}^{Site}$ will be used to refer to these precedences. Because of the mining method used, stopes are also linked to one another by precedences, but contrarily to the site precedences, only the first activity of the stope sequence needs to be completed before the successor stope can be started. The precedence graph $\mathcal{G}^{Stope}$ will be used to refer to these precedences.

The model presented in this article, uses two different time discretizations to reach adequate precision for short-term planning and the foresight needed for medium-term planning while keeping it tractable. Thus the first 3-month period is divided into 12 weeks and the remaining of the planning horizon is divided in 3-month periods (ex: 1 year = 12 one-week periods + 3 three-months periods). Three main indexes are used through the model, s refers to sites, c to crews and t to the time periods. Two other indexes, v and l are used to designate the veins and levels respectively. A list of all sets, parameters, variables and equations follow.

5.4.1 Sets

$\mathcal{E}_{sct}^{Ex}$ and $\mathcal{E}_{sct}^{In}$ represent respectively the set of site/crew pairs that are impossible (exclusive) and possible (inclusive) to complete in the same period t than a given site/crew pair (s, c). These sets are computed through pre-processing procedures that return the minimum length of time between the end and start of all combinations of pairs sites/crew in the precedence graphs $\mathcal{G}_s^{Crew}$ and $\mathcal{G}^{Site}$.

5.4.2 Parameters

Parameters $T_{scs'c'}^{D}$, $T_{scts'c't'}^{S}$ and $T_{scts'c't'}^{SL}$ are also obtained from a pre-processing using the precedence graphs $\mathcal{G}_s^{Crew}$ and $\mathcal{G}^{Site}$. Parameters T_{sc}^{S} and T_{sc}^{C} are both measures of time for a crew in a site but represent different values. T_{sc}^{C} is equivalent to the time actually spent by the specified crew in the site whereas T_{sc}^{S} is the minimum span between the start of the crew's activities in a site and its completion. The difference between these two values comes from the fact that some crew, like the jumbo drill for example, must visit more than one site per shift to be at full capacity. Thus, the actual time spent in each of these sites is lower

$\mathcal{C}_s^H$	:	Set of crews performing haulage activity at site s
$\mathcal{C}_s^f$	:	Set of crews required to perform the first task at site s
$\mathcal{C}_s^e$	:	Set of crews required to perform the last task at site s
$\mathcal{C}_{sc}^P$	:	Crew preceding crew c in site s
$\mathcal{E}^S$	:	Set of all sites $\mathcal{E}^S = \{1, ..., S\}$
$\mathcal{E}^C$	:	Set of all crew types $\mathcal{E}^C = \{1, ..., C\}$
$\mathcal{E}^T$	:	Set of all time periods $\mathcal{E}^T = \{1, ..., T\}$
$\mathcal{E}^L$	:	Set of all levels $\mathcal{E}^L = \{1, ..., L\}$
$\mathcal{E}^V$	:	Set of all veins $\mathcal{E}^V = \{1, ..., V\}$
$\mathcal{E}_l^S$	:	Set of sites located on level l
$\mathcal{E}_v^S$	:	Set of sites located in vein v

$\mathcal{E}_{sc}^D$	:	Set of pairs (s', c') that have to respect a certain delay after the completion of (s,c)
$\mathcal{E}_{sct}^{Ex}$	:	Set of pairs (s', c') that are not feasible in period t if (s,c) are active
$\mathcal{E}_{sct}^{In}$	:	Set of pairs (s', c') that are feasible in period t if (s,c) are active
$\mathcal{P}_s^P$	:	Set of sites preceding site s $(s', s) \in \mathcal{G}^{Site}$
$\mathcal{S}_s^P$	:	Set of stopes preceding stope s $(s', s) \in \mathcal{G}^{Stope}$
$\mathcal{S}_s^A$	:	Set of stopes adjacent to stope s

than the span of the activity. Parameters T_{st}^{start} and T_{st}^{finish} are derived from these values and from the minimum rate. Finally, parameter R_t was used to generalize the formulation and is worth 1 for the short-term periods and 12 for the medium-term.

5.4.3 Variables

The planning variable x_{sct} is used to represent the non-cumulative completion of activities for each time period as a continuous variable. The two other binary variables χ_{sct} and γ_{st} are mostly used to enforce precedences and limit activity duration.

5.4.4 Objective

The objective of the model presented in Equation (5.1) is to maximize the NPV of all activities executed over the horizon. Thus the objective function is simply the summation of the discounted values of the activities performed for each time period.

$$Max \qquad \sum_{s=1}^{S}\sum_{c=1}^{C}\sum_{t=1}^{T} \delta_t C_{sc} x_{sct} \tag{5.1}$$

A_{ct} : Available crews of type c at time t (units)
C_{sc} : Cash flow associated with the activities of crew c at site s
T^P : Number of work hours available per time period (hours)
T_s^{Tot} : Maximum time span in weeks of all activities in site s (weeks)
$T^D_{scs'c'}$: Minimum time span in hours between the end of crew c activities at site s and the start of crew c' at site s' (hours)
$T^S_{scts'c't'}$: Minimum waiting time in hours for the start at site s' of crew c' at period t' if site s, crew c was active at period t (hours)
$T^{SL}_{scts'c't'}$: Minimum waiting time in hours for the start at stope s' of crew c' at period t' if stope s, crew c was active at period t (hours)
T^S_{sc} : Minimum time span in hours needed for crew type c to process it's activity at site s (hours)
T^C_{sc} : Number of work hours needed from crew type c to process it's activity site s (hours)
T^{start}_{st} : Minimum time period where activities at site s can be started and still be active at time t
T^{finish}_{st} : Maximum time period where activities at site s can still be active if started at time t

Q_s : Rock tonnage in site s (tonnes)
Q^M : Maximum possible tonnage extraction in the mine for one time period (tonnes)
Q^L_l : Maximum possible tonnage extraction in level l for one time period (tonnes)
Q^V_v : Maximum possible tonnage extraction in vein v for one time period (tonnes)
Q^O_t : Minimum ore tonnage to be extracted in period t (tonnes)
R_t : Length of period t (weeks)
δ_t : Discount factor for the objective at time period t

5.4.5 Constraints

Modeling Constraints

Constraints (5.2) limit the activity to be completed at most once. The inequality can be changed to an equality in cases where an activity has to be completed. Further discussion on this will follow in the result section. Constraints (5.3) and (5.4) respectively link binary variables χ_{sct} to variables x_{sct} and γ_{sct}. Constraints (5.5) make sure that if an activity is started, it is completed within a certain number of periods, assuring a minimum completion rate. Constraints (5.6) force any sites where activities are started to complete all of them unless they are started at the last period. The chosen interval of the summation helps in the branching process by making the model tighter. The constraints are also relaxed on the last period to allow the model to start activities in this last period that would in real-

x_{sct}	:	Fraction of work from crew c executed at site s during time period t
χ_{sct}	:	Binary variable indicating whether or not crew c is active at site s during time period t
γ_{st}	:	Binary variable indicating whether or not any of the crew is active at site s during time period t

life be completed on the following weeks or months. Constraints (5.7) ensure that if an activity completion spans over more than 3 time periods, it is completed at the maximum rate between the beginning and end period.

$$\sum_{t \in \mathcal{E}^T} x_{sct} \leq 1 \qquad \forall s, c \tag{5.2}$$

$$x_{sct} - \chi_{sct} \leq 0 \qquad \forall s, c, t \tag{5.3}$$

$$\chi_{sct} - \gamma_{st} \leq 0 \qquad \forall s, c, t \tag{5.4}$$

$$\sum_{t \in \mathcal{E}^T} \gamma_{st} \leq T_s^{Tot} \qquad \forall s, c \tag{5.5}$$

$$\chi_{sc't} - \sum_{t'=T_{st}^{start}}^{T_{st}^{finish}} x_{sc''t'} \leq 0 \qquad \forall s, c' = \mathcal{C}_s^f, c'' = \mathcal{C}_s^e, t \in \mathcal{E}^T \setminus \{T\} \tag{5.6}$$

$$T^P \chi_{sct-1} - T_{sc}^S x_{sct} + T^P \chi_{sct+1} \leq T^P \qquad \forall s, c, t \in \mathcal{E}^T \setminus \{1, T\} \tag{5.7}$$

Tonnage Constraints

Constraints (5.8) limit sum of haulage activities to the mine limit, often corresponding to the shaft or ramp haulage limit. Constraints (5.9) and (5.10) impose similar limits on haulage activities respectively for each level and vein. These level and veins limits usually correspond to the haulage capacity for certain sectors of the mine, or an estimate limit to avoid congestion delays in a section's operations. Constraints (5.11) enforce a lower bound on the amount of ore extracted for each time period. These lower bounds correspond to the minimum ore tonnage needed to keep the mill active during any time period.

$$\sum_{s\in\mathcal{E}^S} x_{sc't}Q_s \leq Q^M \qquad \forall c' \in \mathcal{C}_s^H, t \tag{5.8}$$

$$\sum_{s\in\mathcal{E}_l^S} Q_s x_{sc't} \leq Q_l^L \qquad \forall l, c' \in \mathcal{C}_s^H, t \tag{5.9}$$

$$\sum_{s\in\mathcal{E}_v^S} Q_s x_{sc't} \leq Q_v^V \qquad \forall v, c' \in \mathcal{C}_s^H, t \tag{5.10}$$

$$\sum_{s\in\mathcal{E}^S} x_{sc't}Q_s \geq Q_t^O \qquad \forall c' \in \mathcal{C}_s^H, t \tag{5.11}$$

Time Constraints

Constraints (5.12) make sure that the sum of the time spend by all crews in a period is lower than the available time in the period. Constraints (5.13) make sure that the span associated with the completion of any activity in a time period is lower than the available time in the period. One may point out that Constraints (5.12) are included in Constraints (5.13), but the presence of Constraints (5.12) makes the formulation much tighter since it provides a stronger link between variables x_{sct} and χ_{sct} than Constraints (5.3) in cases where T_{sc}^S is greater than $T^P R_t$. Constraints (5.14) limits the amount of time needed by any crew type for a time period to the number of work hours available for the type.

$$T_{sc}^S x_{sct} - T^P R_t \chi_{sct} \leq 0 \qquad \forall s, c, t \tag{5.12}$$

$$\sum_{c\in\mathcal{E}^C} T_{sc}^S x_{sct} \leq T^P R_t \qquad \forall s, t \tag{5.13}$$

$$\sum_{s\in\mathcal{E}^S} T_{sc}^C x_{sct} \leq A_{ct} T^P R_t \qquad \forall c, t \tag{5.14}$$

Precedences Constraints

Constraints (5.15) and (5.16) enforce respectively the precedence relations between the activities of graphs $\mathcal{G}_s^{Crew}$ and $\mathcal{G}^{Site}$. Constraints (5.17) make sure that a pair of ancestor/descendant in $\mathcal{G}^{Site}$ that are separate by too much time cannot be completed in the same time period and help the branching process by linking integer variables together. Constraints (5.18) on the other hand make sure that the time spent on a pair of ancestor/descendant in $\mathcal{G}^{Site}$ in a time period is lower than the available time in the period minus the minimum time between them. Those are necessary to ensure that the model does not plan for the execution of an ancestor and its descendant simultaneously. Without this constraint a feasible solution could require to work the equivalent of a whole period in a site and it's successor in the same time period. Similarly, Constraints (5.19) ensure that the time distance between a pair of ancestor/descendant in $\mathcal{G}^{Site}$ is respected among activities completed in different time periods while tightening the formulation.

$$\sum_{t'=1}^{t} x_{sc't'} - \chi_{sct} \geq 0 \qquad \forall s, c, c' \in \mathcal{C}^{P}_{sc}, t \tag{5.15}$$

$$\sum_{t'=1}^{t} x_{s'c't'} - \chi_{sc''t} \geq 0 \qquad \forall s, s' \in \mathcal{P}^{P}_{s}, c' \in \mathcal{C}^{f}_{s}, c'' \in \mathcal{C}^{L}_{s'}, t \tag{5.16}$$

$$\chi_{sct} + \chi_{s'c't} \leq 1 \qquad \forall s, c, t, \{s', c'\} \in \mathcal{E}^{Ex}_{sct} \tag{5.17}$$

$$T^{D}_{scs'c'}\chi_{sct} + T^{D}_{scs'c'}\chi_{s'c't} + T^{S}_{sc}x_{sct} + T^{S}_{s'c'}x_{s'c't} \leq T^{P}R_{t} + T^{D}_{scs'c'} \qquad \forall s, c, t, \{s', c'\} \in \mathcal{E}^{In}_{sct} \tag{5.18}$$

$$T^{L}_{scts'c't'}\chi_{sct} + T^{L}_{scts'c't'}\chi_{s'c't'} + T^{S}_{sc}x_{sct} + T^{S}_{s'c'}x_{s'c't'} \leq T^{P}(R_{t} + R_{t'}) + T^{L}_{scts'c't'} \qquad \forall s, c, t, \{s', c', t'\} \in \mathcal{E}^{D}_{sc} \tag{5.19}$$

Stopes Constraints

Constraints (5.20) make sure that the first activity of a stope can only start when the first activity of its predecessor in $\mathcal{G}^{Stope}$ is completed and Constraints (5.21) forbid any activity to happen in an adjacent stope during a stope's curing time.

$$\sum_{j=1}^{t} x_{s'c'j} - \chi_{sc''t} \geq 0 \qquad \forall s, s' \in \mathcal{S}^{P}_{s}, c' \in \mathcal{C}^{f}_{s}, c'' \in \mathcal{C}^{F}_{s'}, t \tag{5.20}$$

$$T^{SL}_{scts'c't'}\chi_{sct} + T^{SL}_{scts'c't'}\chi_{s'c't'} + T^{S}_{sc}x_{sct} + T^{S}_{s'c'}x_{s'c't'} \leq T^{P}(R_{t} + R_{t'}) + T^{SL}_{scts'c't'} \qquad \forall s, c, t, \{s', c', t'\} \in \mathcal{S}^{A}_{s} \tag{5.21}$$

Definition Constraints

Constraints (5.22) and (5.23) define the non-negative and binary nature of the variables.

$$x_{sct} \geq 0 \qquad \forall s, c, t \tag{5.22}$$

$$\chi_{sct}, \gamma_{sct} \in \{0, 1\} \qquad \forall s, c, t \tag{5.23}$$

As seen from the constraints, the model creates preemptive schedules with certain limitations for example on the duration of each activity. The continuous variables are mostly chosen to produce schedules that are less dependent on the time discretization used. The advantage of using continuous variables in situations similar to ours was demonstrated in [74]. The model also includes some constraints that are redundant with others, like Constraints (5.12) and (5.13), to make the model tighter. Those constraints were added following early tests that showed the complexity of the problem and their addition proved to speed up the resolution. The general formulation was made to be as tight as possible.

5.5 Results

In order to test the tractability of our model, a dataset based on values taken from an operating Canadian gold mine was used. It includes 338 possible workplaces, where 842 activities must be completed by 10 types of specialized crew or equipment. Table 5.3 gives an overview of the principal characteristics of the dataset used for the experimentation. Although the costs and profits used for the tests presented further in this article are not the ones actually used at the mine, they represent a realistic approximate.

Table 5.3 Dataset Characteristics

	Characteristics
Mining Methods	Long-Hole and Cut-and-Fill
Nb Veins	4
Nb Levels	4
Development Crews	Jumbo Drill, Raise, Track, LHD
No Development sites	228
Length of Development (m)	7374
Production Crews	Cable drill, Cable Ciment, Production Drill, Loader, LHD, Barricade, Backfill
No Production sites	110
Ore Tonnage (t)	402 500

All of the tests presented below were implemented using IBM ILOG CPLEX Optimization Studio version 12.8.0.0 branch-and-cut algorithm with up to eight threads. The computer used to run the tests was an Intel(R) Core(TM) i7-4770 CPU @ 3.40GHz with 16 GB of RAM. The optimality precision tolerance, the maximum optimality gap at which a solution is considered optimal, was set to 0.1% and a time limit for all test was set to one hour. This section will first present the results of the application of our model to the original model and then improvements made to increase applicability and tractability.

5.5.1 Initial Model

The datasets referred to here as scenarios one to five, in order of complexity, were used to test our model in its initial version. The unmodified dataset corresponds to Scenario 2, with Scenario 1 being a variation where development lengths were randomly increased by 0 to 40%. Scenarios 3, 4 and 5 are datasets where veins with stopes and developments were added to the initial dataset. Table [5.4] presents the result of the implementation of our model to the different scenarios in terms of variables and constraints. A procedure based on the computation of the earliest start of each task from precedence graph $\mathcal{G}^{Site}$ was used to reduce the size of the problems.

Table 5.4 Variables and Constraints of Scenarios 1 to 5

	S1	**S2**	**S3**	**S4**	**S5**
Continuous Variables	1314	1703	2877	3860	3981
Binary Variables	2392	3080	5231	7060	7286
Constraints	24323	31307	52048	68240	71033

Table [5.5] shows the numerical results of the implementations. The first and second lines of the table show the objective function value of the best integer solution found and the relative gap between this solution and the best upper bound available at the termination. The next two lines indicate the time needed to solve the initial linear relaxation at node zero of the branching tree and its objective value. The following three lines refer respectively to the number of nodes processed during the branching procedure, the number of nodes remaining at the end of it, and the number of integer solutions found along the way. The first three lines of the time section indicate how much time was needed to find a solution respectively within 5%, 1% and 0.1% of the optimal solution. The final lines then indicate the time needed to find the final solution and the total computation time.

From Table [5.5], it can be observed that scenarios 4 and 5 could not be solved to optimality. Scenario 4 was stopped by the time limit and Scenario 5 was stopped because the size of its branching tree exceeded the available memory after 40 minutes of computation. When looking at the LP Relaxation results, it can be observed that the LP relaxation of the problem is a bad estimate of our problem. If for the smaller datasets the integrity gap (the difference between the objective value and the LP relaxation value) is reasonable at less than 20%, it grows steadily with the size of the problem to reach near to 80%. [74] illustrates with an example how the relaxed solution of a problem with similar properties is far from the integer solution. The branching results show that many integer solutions are found along the branching tree, which shows that the feasibility is not the most constraining part of the problem. The time section also corroborates this theory with several good solutions found early in the resolution. In most cases, the longest time is spent proofing the optimality rather than finding the solutions. This combination of a great number of solutions found and a lot of time spend on optimality proofing is an indicator that there is a lot of symmetry in our problem, caused by many similar options in planning. CPLEX comes with algorithms to help break symmetry in problems, and even with the most aggressive symmetry breaking setting no difference in solving time was noticed. This is also why many efforts were put on trying to make the formulation as tight as possible.

Table 5.5 Initial results of scenarios 1 to 5

Value	**S1**	**S2**	**S3**	**S4**	**S5**
Objective	1.60E+07	2.19E+07	2.56E+07	2.54E+07	2.61E+07
Gap (%)	0.1	0.1	0.1	5.1	0.6
LP Relaxation					
Time	0.02	0.02	0.1	0.22	0.22
Value	1.94E+07	2.65E+07	3.84E+07	4.59E+07	4.68E+07
Branching					
Processed	5467	122305	1133175	615295	656620
Remaining	925	40386	325682	264139	295912
Nbr of Solution	31	30	61	41	23
Time (s)					
5%	0.76	1.96	12.7	-	-
1%	0.76	1.96	12.7	-	-
0.1%	0.76	2.21	12.8	-	-
Best Solution	5.06	12.0	103.6	251.5	71.1
Total	5.40	140.2	1946.4	3600	2360.4

5.5.2 Improved Model

From the observation of the original results, many conclusions were made about possible improvements. First, in its original formulation, the problem grows quickly to become intractable with the addition of more zones and secondly, the schedules produced were not realistic. The reason is that in the solutions produced, the model would schedule all the possible stopes as early as possible and then makes the least possible development and push it as far as possible to maximize the NPV. This gave solutions where, in the first weeks, no activities are scheduled and then only the necessary developments are performed. This kind of schedule is for obvious reasons not realistic since the different equipment available in the mine are left idle for long periods of time whereas in real life, they would be assigned to developments. This aversion of the model to development also creates solutions where no developments are done in preparation for stopes to be extracted outside of the resolution time frame. Now the reason why the NPV works correctly for many of the available models in the literature and not for ours is that these models use as input a pre-defined set of tasks that have to be completed rather than a range of possible tasks to choose from, like ours. This forces the models to complete necessary developments even if they represent a loss in the short-term. In our case though, the model is not forced to complete these tasks.

To fix these problems, a modification was made to the objective. Instead of using the NPV, the absolute values of the discounted profits and expenses were used. The idea behind it is that this change will mainly move the non-critical developments earlier in the planning while keeping the production, and the critical developments leading to it, mostly unchanged. The reasons for it are that first, in a precious metal mine, the production revenues are typically much higher than any development cost. In our dataset, the average cost of a development is less than 5% of the average profit of a stope. Moreover, development is the main limiting factor for production since any available production is extracted as soon as possible. Thus, the model tries to complete production activities as early as possible and since all development activities are predecessors to a production activity in $\mathcal{G}^{Site}$, all developments on the critical path to the production are also completed as early as possible. Using the absolute value of the NPV does not change this fact, and so, it will mostly affect the non-critical developments that will be completed earlier than later.

This effect is actually desirable for two main reasons. First, in underground mines, costs are computed in $\$/m$ and are mostly due to personnel, equipment and consumables. An exception to this rule would be for the main developments, like shafts for example, that are much more expensive to complete, but the scheduling of these developments are decided at the highest level of planning and thus are not included in our model. For a given crew, there is very little difference in cost between possible assignments. Thus, any planning with maximum equipment usage will produce the same development cost and there is no gain in delaying its execution. Secondly, activities in underground mines are subject to a lot of uncertainty and it is common practice for planners to start developments sooner than later to palliate to unpredictable delay that could happen in the execution of the activities. This practice helps produce more robust solutions. On a mathematical perspective, such a model should also be simpler to solve since instead of having two conflicting objectives; pushing development as far as possible and bringing production as early as possible, the objective consists of doing the maximum in the time horizon with a priority on the most valuable stopes.

Table 5.6 shows the results of the application of the modified model using the same format as used in Table 5.5. First, it can be noticed that Scenario 5 was solved to optimality in less than 700 seconds when it could not be solved with the previous formulation, and Scenario 4 reached a slightly smaller, but still not acceptable, optimality gap. The LP Relaxation section shows solving times of the same order but smaller integrity gaps, all of them under 60%. The number of branches necessary to proof optimality is also decreased in all of the scenarios. The most important result though is the decrease in total solving time for all scenarios. Many different combinations of parameters were tested in order to improve the solving efficiency and it was found that the "Hidden Feasibility" emphasis in branching implemented in CPLEX gave the best results. Table 5.7 gives an overview of the results of the implementation with the optimal parameters. Similar parameters were tested with the original model but did not improve the solving time, which is why the default parameters were used for the comparison. The main takeaway of this last table is that with the right objective and solving parameters, all of the scenarios could be solved to optimality within 1 hour of computations.

Table 5.6 Improved results of scenario 1 to 5

Value	**S1**	**S2**	**S3**	**S4**	**S5**
Objective	2.58E+07	3.16E+07	3.64E+07	3.62E+07	3.66E+07
Gap (%)	0.1	0.1	0.1	4.7	0.1
LP Relaxation					
Time	0.02	0.03	0.19	0.38	0.38
Value	2.84E+07	3.60E+07	4.95E+07	5.77E+07	5.84E+07
Branching					
Processed	3367	8704	16670	133239	13122
Remaining	134	1642	214	34834	1112
Nbr of Solution	74	63	65	162	46
Time (s)					
5%	1.38	3.25	214.9	-	395.7
1%	1.53	3.25	214.9	-	395.7
0.1%	1.62	6.26	252.3	-	395.7
Best Solution	4.98	26.4	252.5	3582.5	697.2
Total	4.98	26.5	255.6	3600	697.2

Table 5.7 Improved results of scenario 1 to 5 with parameter optimization

Value	**S1**	**S2**	**S3**	**S4**	**S5**
Objective	2.58E+07	3.16E+07	3.64E+07	3.63E+07	3.66E+07
Gap (%)	0.1	0.1	0.1	0.1	0.1
Total time (s)	3.39	15.8	216.9	3032.8	1177.5

Figure 5.1 clearly illustrates the mathematical advantages of the modified objective. It is a graphical representation of values of χ_{sct} taken from the integer and relaxed solutions of the initial (NPV) and improved (ABS) model. The sites are all stopes following one another

from the same vein and the crew chosen is a drilling team. The time periods represented by each column in the figure are the four last 3 months time periods. It can be noticed first that the integer solutions for both models are the same. The relaxation of the improved model is also very similar to the solution with all variables being already integers. Then the relaxation of the initial model shows many fractional values. This example clearly illustrates why the modified problem takes less branching to solve to optimality. These sites were chosen because they best express the advantages of the modified objective, but many other variables from the relaxation were fractional. Still, when looking at the whole problem, 7.2% of non-zero χ_{sct} variables were integers in the initial relaxation whereas 28.1% of them were integers in the modified relaxation, which clearly shows that the trend seen in the previous example can also be seen in the whole model.

	Integer (ABS)				Integer (NPV)				LP Relaxation (ABS)				LP Relaxation (NPV)			
Stope 1	1	0	0	0	1	0	0	0	1	0	0	0	0.4	0	0	0
Stope 2	1	0	0	0	1	0	0	0	1	0	0	0	0.4	0.3	0.1	0.1
Stope 3	1	1	0	0	1	1	0	0	1	0	0	0	0.4	0.4	0.1	0
Stope 4	0	1	0	0	0	1	0	0	0	1	0	0	0	0.7	0.1	0.1
Stope 5	0	1	0	0	0	1	0	0	0	1	0	0	0	0.7	0.1	0.1
Stope 6	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0.9	0.1
Stope 7	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0.9	0.1
Stope 8	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0.9	0.1
Stope 9	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0.1	0.9
Stope 10	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0.9
Stope 11	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0.9

Figure 5.1 Comparison of χ_{sct} values for initial and improved model

The advantages of the modified model in a computational perspective were proven in the previous paragraphs, and the practical advantages will now be covered. Table 5.8 indicates for each of the tested scenarios the practical results. The first section of the table shows the total NPV as well as the discounted value of the development and production part of the solution for the initial model. The number of completed sites is then shown on the last line. The second section shows first the value of the modified objective for each solution in line ABS. The solution was then used to compute the actual NPV and the results with the detailed value of discounted development and production are displayed. The number of completed sites is then presented on the last line. The final section presents the difference between the value of discounted productions in percentage and the difference in number of completed sites for the initial and modified models.

When looking at the NPV of each scenario for the initial and modified model, it is clear that the NPV of the solutions of the initial model are always higher than the modified one.

Table 5.8 Practical comparison of initial and modified objective

Initial	**S1**	**S2**	**S3**	**S4**	**S5**
NPV (M$)	16.03	21.87	25.62	25.42	26.11
Development (M$)	-3.26	-3.32	-3.53	-3.50	-3.32
Production (M$)	19.28	25.19	29.15	28.92	29.43
Nb completed	148.0	210.0	227.0	226.0	221.0
Modified					
ABS (M$)	25.74	31.64	36.39	36.30	36.63
NPV (M$)	12.82	18.74	22.57	21.72	22.24
Development (M$)	-6.46	-6.45	-6.91	-7.29	-7.19
Production (M$)	19.28	25.19	29.48	29.01	29.43
Nb completed	204.1	283.1	325.5	337.8	324.5
Comparison					
Production (%)	-0.0219	0.0002	1.4823	0.4221	0.0005
Δ Nb completed	56.1	73.1	98.5	111.8	103.5

But when taking a closer look, one can notice that the amount of development made by the modified model is always much higher. This is also confirmed by the number of completed sites that is also higher for the modified model. As explained before, since the development costs are mostly fixed and would be spent in any way by keeping the equipment active, the modified model solutions are much closer to a realistic schedule by completing as much as possible during the time available. For the production part, where the profit is made, we notice similar numbers with the value of the modified model being slightly higher for most of the scenarios. This proof that using that absolute value of the objective creates solutions for production that are very similar to what the standard NPV would produce. The fact that most of the production value for the modified model are higher also show that by trying to avoid development expenses, the model can produce solutions that do not get the most production done during the time allowed. The only exception to this is for the first scenario, but the difference being smaller than the optimality gap, it can be considered as negligible.

5.5.3 Application-Oriented Model

In order to test the model with scenarios that are closer to what a normal usage would be by the planers of an underground mine, a new set of scenarios were developed. Scenarios 6 to 9 are all identical to the Scenario 2, with the exception that the time horizon considered changed and different sites were forced to be completed. This is done by changing constraints 5.2 to an equality for the sites that must be completed. Scenario 6 uses a time horizon of 1 year and 3 months, as scenario 2, but forces the completion of the development of a vein that was not included in the solution of scenario 2. Scenario 7 uses a time horizon of 1 year and 9 months (twelve week periods and six “3 months” periods) and Scenario 8 uses a time horizon of 2 years and 3 months (twelve week periods and eight “3 months” periods). Finally, Scenario 9 uses a time horizon of 3 years and 3 months (twelve-week periods and twelve “3 months” period) and forces the completion of all the sites in the dataset. Table 5.9 shows the characteristics in terms of variables and constraints of each scenario.

Table 5.9 Variables and Constraints of Scenarios 6 to 9

	S6	**S7**	**S8**	**S9**
Continuous Variables	1714	2891	4401	7725
Binary Variables	3102	4875	7040	11710
Constraints	31402	72122	140971	320181

Table 5.10 shows the results of the application of the modified model to scenarios 6 to 9 in the same format as the one used in previous tables. The last column, “S9 (NPV)” being the application of the original model to Scenario 9. The results show that fixing the completion of certain sites greatly improve the solving time for the model. Even for scenarios that had horizons up to two times longer than the original one, the optimal solution could be found within 12 minutes. This is due in part to a smaller integrity gaps and good solution found earlier in the branching as the time section shows. As for Scenario 9, even if the optimality could not be reached in both cases, the modified objective produced much better results. This scenario is very similar to a long-term model considering that the time horizon covers more than three years and that it plans for the executions of all the activities in the dataset. First, it produced a solution very close from the optimality tolerance and even more important, it found 109 feasible solutions where the original version did not find any. Moreover the NPV value of the best solution found by our modified model was 1.01E+08, that is less than 1% away from the best-known bound on the original problem formulation. This comparison shows very promising results for our modified objective for an application to long-term models since our modified objective could produce solutions of good quality much faster than a regular formulation. These solutions could then be used as warm starts for branching or starting points for heuristics applied on other models.

5.5.4 Integration Comparison

Tries were made to measure the monetary benefits of using an integrated model instead of using two separate planning for short- and medium-term. To do so, a first solve was made using a time horizon of twelve one-week periods and using the absolute value objective. The solution of this first solve was then used to fix variables for the first twelve weeks in a second solve with 12 one-week periods followed by 4 three-months periods. The solutions from

Table 5.10 Improved results of scenario 6 to 9

Value	**S6**	**S7**	**S8**	**S9**	**S9 (NPV)**
Objective	2.94E+07	6.19E+07	8.62E+07	1.16E+08	-
Gap (%)	0.1	0.1	0.1	0.2	-
LP Relaxation					
Time	0.03	0.08	0.16	0.38	0.72
Value	3.49E+07	7.37E+07	1.03E+08	1.17E+08	1.02E+08
Branching					
Processed	1963	81425	20430	48325	5849
Remaining	18	8694	4770	17624	1400
Nbr of Solution	36	133	89	109	0
Time (s)					
5%	2.13	31.0	48.6	-	-
1%	2.13	185.1	129.9	-	-
0.1%	2.13	185.1	132.1	-	-
Best Solution	9.71	662.3	235.4	3584.7	-
Total	10.4	663.1	289.3	3600	3600

these partly fixed models were then compared to the solutions of the original models but the results were not as good as expected. For all of our scenarios, the difference in the objective values was between 0.2% and 1%. The small gain in value can probably be explained in part by our modified objective that diminishes the aversion of typical models to plan for extra developments. Nevertheless, the integration could yield larger benefits on other datasets and still represents a major benefit from the planners perceptive. Grouping two planning levels together reduces the time spent on each of them and does not require the planner spend time splitting medium-term objectives in smaller portions to use as an input for short-term planning.

5.6 Conclusion

This article presented a model for integrated short- and medium-term underground mine planning. It uses continuous variables to produce solutions that are realistic and not dependent on the time discretization used. A modified objective was then introduced to fix the flaws of the original one, namely the difficulty to solve and the low usage of equipment. The advantages and applicability of the modified objective were then demonstrated with mathematical and practical demonstrations. This modified objective showed promising results especially for potential applications to long-term models. A final set of scenarios was then tested with this modified objective to demonstrate the application possibilities of such a model. Many of the underground mine planning models share common characteristics like the long chains of precedence with profitable activities coming after many expenses. These similarities lead the authors to think that the application of the modified objective to existing long-term models could lead to improvement in their tractability. Further research will be needed to explore these possibilities.

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CHAPITRE 6 ARTICLE 3: SHORT- AND MEDIUM-TERM OPTIMIZATION OF UNDERGROUND MINE PLANNING USING CONSTRAINTS PROGRAMMING

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6.1 Abstract

For the past few years, the mining industry has seen a lot of operational changes. The digitalization and automation of many processes have paved the way for an increase in its general productivity. In keeping with this trend, this article presents a novel approach for optimizing underground mine scheduling for short- and medium-term. It is a well-known problem similar to the Resource-Constrained Project Scheduling Problem, with some modifications. The model uses Constraint Programming principles to maximize the Net Present Value of a mining project. It plans work shifts for up to a year ahead, considering specialized equipment, backfilling and operational constraints. Results from its applications to datasets based on a Canadian gold mine demonstrate its ability to find optimal solutions in a reasonable time. A comparison with an equivalent Mixed Integer Programing model proves that the Constraint Programming approach offers clear gains in terms of computability and readability.

6.2 Introduction

Underground mines are a unique environment with their own challenges. Rock mechanics, dewatering, ventilation and the choice of one of the many mining methods are only a few examples of the technical aspects that have to be taken into consideration. In order to consider all these challenges while keeping focus on the global profitability, many different levels of planning are used over the course of a mine, each of them considering certain aspects of underground mining. The planning of an underground mine generally starts by creating a “life of mine.” This is usually done at the very beginning of a mine and is revised once every few years with regards to new information gathered from exploration drilling or accordingly with the pace of development. Through this, the general shape of the developments needed to access the ore body is drawn and the ore body itself is split in approximative stopes, i.e. unitary subsection of the ore body, with generic size and shape. Long-term planning is based on this life of mine and plan activities with a very low granularity over periods of one year. The focus of this exercise is mostly on the economical aspect, with most of the technicality being considered through rough approximations. The first year of this planning is then used to create a medium-term planning, which will generally be revised every few months. At this stage, activities are planned with more precision over periods ranging typically from one to three months accordingly to the general objectives defined by the long-term planning. Using the first periods of medium-term planning, the short-term planning is then created with periods of one or two weeks and revised every few weeks. This last level of planning is used daily by planners and foremen to decide the order of activity through the next shifts and the dispatch of equipment between these tasks.

Many specialized crews and equipment are necessary in an underground mine depending on the mining method, type of developments and rate of production. These crew can either work one after another or semi-simultaneously on the advance of a site. What is meant by semi-simultaneously is in a rapid succession within a single work shift and repeated for multiple days. The former generally happens in stopes, where a lot of work is needed to prepare a single blast that will provide haulage material for many shifts, whereas the former usually happens in the developments of galleries where a series of activity needs to be repeated for as many blast as necessary. Blasts can be seen as a base unit of distance in vertical or horizontal developments in underground mines. The length of excavated rock they produce is fixed for a given type of heading as well as the amount of work required. For safety reasons, blasts can only happen in between shifts (generally twice a day), so the time required to excavate a gallery is more dependent on the number of blast than the sum of working time since it generally takes less than a shift to complete the work sequence leading to a blast. In the case of stopes, many mining methods require a "backfilling" as a last activity. This activity consists in filling the empty stope with a mix of rock and concrete in order to stabilize neighboring stopes. A delay of two to three weeks then has to be allowed for the concrete to solidify before anything can happen in adjacent locations. All of these concepts (e.g. backfilling, fixed-time blast, specialized crew) are unrelated to open-pit mining, which makes the underground planning problem very different.

The point of this article is to develop a model that would integrate short- and medium-term planning for underground mines into a single model. The model has to create planning over a horizon of more than a year in order to consider long-term objectives, while being detailed enough to be used at the short-term level. Moreover, the model has to be scalable to allow for a frequent re-optimization of the problem accordingly to operation changes and unplanned events. The application of such model to real-world mining operations would yield many benefits. First, it would produce automatically optimal solutions to a problem that is still solved by hand in most of the mining industry. Second, it would address the loss of optimality coming from successive resolution of the different planning levels by finding the global optimal solution to short- and medium-term planning. Third, it would require a lot less time and would guarantee a much faster response time for the planner to produce these updated schedule every time a change occurs and a new planning needs to be created.

This article will present a model of Constraint Programming (CP) that address the points mentioned here and compare its capacity to a more traditional Mixed Integer Programming (MIP) model. It is a novel approach at solving this problem since, as it will be seen in the next section, all of the examples available in the literature are MIP models. Nevertheless, CP has proven its value for many types of problems, including the closely related Resource-Constrained Project Scheduling Problem.

6.3 Literature Review

There are many examples of optimization in the mining industry as found in [15] and [16] reviews of the literature of optimization in natural resources and mining. Most of these developments though are specific to open pit mining, as reviewed in [13] among others. Even if developments are fewer, the reader can find in [19] an overview of recent developments and opportunities in underground mine optimization.

More specifically in planning for underground mines, some models are available for the optimization of long-term planning. In [36], the authors present a model for the optimization of a mining complex including underground and open-pit mines. More recently, [40] describe a method for optimizing a similar problem comprising many underground and open-pit mines while taking into consideration geological uncertainty. On a more technical perspective, [37] present a generalized model and solving procedure based on column generation for solving a variation of the Resource Constrained Project Scheduling Problem (RCPSP) which aims at maximizing the discounted cash flow (RCPSPDC). The model is then tested on a long-term scheduling problem from an underground mine. Similarly, [41] demonstrate that the well know methodology presented in [42] to solve LP relaxations of open-pit problems can be applied to solve relaxations of a much broader category of problems like the RCPSP, including underground mine planning problems.

Some articles present the solutions to the integration of other aspects of underground mines. For example, [46] describe a model for the simultaneous optimization of stopes design and scheduling. In a similar fashion, [38] present a model for the optimization of underground mine schedule while considering the cut-off grades i.e. the minimum ore content at which rock is considered to be ore. The authors of [39] later proposed a stochastic variation of this model. The model presented in [47] also optimize the schedule while taking into consideration variable stope size, but does so by using linear approximations of grade versus tonnage curves for the different stopes.

Short- and medium-term models have also been developed but tend to but more application specific as for these kinds of time horizons, many site specific or mining method specific constraints have to be added. The authors of [45] describe a model and a heuristic for the scheduling of activities at LKAB's Kiruna iron ore mine. A model for integrated short- and medium-term planning can also be found in [49] but its application is limited to a conceptual 30 stopes dataset. Another site specific application is described in [51], where the authors describe the results of the application of an optimization model the planning of the final two years of activity at the Lisheen zinc mine in Ireland. The model displayed in [74] allows for the optimization of short-term planning and is tested on a dataset based on a Canadian gold mine. The model from [75] uses a variable time discretization to extend the solvable planning horizon for the same dataset.

As mentioned before, the problem of mine scheduling as many similarities with the RCPSP. The field of constraint programming as proved in the past to be very effective for this class of problem. To cite one among others, [69] show the advantage of CP models over classic MIP models for a variation of the RCPSP called RCPSP/max-cal, where time lags and resource calendars have to be respected while minimizing the total makespan. To the best of the authors' knowledge, the only other underground mining application of CP in the literature is [54], which propose a model for optimal dispatch of equipment for time horizons of less than 72h. For a complete overview of the CP solver used for this article as well as examples, the reader is referred to [56].

6.4 Model

The model uses six different index types. The first one s is used to designate sites. The term site is used to designate any location where an activity can happen. Long galleries are split in segments for each intersection so that a site represents a single piece of tunnel without any branches. Long galleries without intersections are also split in many sites in order to allow for their development to be segmented in a few parts. Index a represents the different activities happening in each site. The next index, c, refers to the different types of specialized crew or equipment working underground e.g. production drilling. Next are indexes l and v that respectively represents levels and veins. Finally, t refers to production periods, used only for the mill feed constraints. The base unit of time used in this model is the work shift (typically

10 hours), so that all durations are rounded up to the next shift. Shifts make a natural unit of time in underground mines since all series of activities have to be ended with a blast, and blast can only be done in between shifts. Hence, even if activities in a gallery are finished in a fraction of a shift, its successor gallery won't start before the next shift. The model presented in this article was built using DOcplex.CP Python API from IBM. The syntax for the function and variables was taken from its documentation.

6.4.1 Sets

Four groups of sets are present in the model: A referring to activities, C referring to crews, P referring to predecessors or adjacent stopes and S referring to sites.

$\mathcal{A}_s^F$	:	First activity at site s
$\mathcal{A}_s^L$	:	Last activity at site s
$\mathcal{A}_s^H$	:	Haulage activity at site s
$\mathcal{A}_{sa}^P$	:	Predecessor activity to activity a at site s
$\mathcal{C}_{sa}$	:	Crews needed for activity a at site s
$\mathcal{P}_s$	:	Predecessor sites of site s
$\mathcal{P}_s^{Adj}$	:	Adjacent stopes of stope $s \in \mathcal{E}^{Stope}$
$\mathcal{P}_s^{Stope}$	:	Predecessor stopes of stope $s \in \mathcal{E}^{Stope}$
$\mathcal{S}_l$	:	Sites located on level l
$\mathcal{S}_v$	:	Sites located in vein v
$\mathcal{S}^B$	:	Sites requiring backfill
$\mathcal{S}^O$	:	Sites containing ore
$\mathcal{S}^{Stope}$	:	Sites that are stopes

6.4.2 Parameters

Parameter A_c, representing the available crew of each type, is expressed as a percentage to ease the representation of fractional usage. For example, if two crews of type c were available for a scenario, A_c would equal 200. This is made necessary by the fact that in underground mine planning, crew are typically assigned to a fixed number of sites until their completion. For example, each production drilling crew will be assigned 3 sites, which correspond to the number of blasts one crew can complete in one day (2 shifts). Assigning crews to fewer sites would significantly reduce their productivity since a delay has to be respected after a blast before workers can return in a site. Thus, having crew cycling through two sites or less would greatly diminish their working time. Assigning a crew to more sites, on the other hand, would slow down the development rate of each of them and dilute the development effort. Parameter U_{sac} represents the percentage of a crew required by an activity in a site. Coming back to our example with the development drilling crew, each activity where it is required would have U_{sac} for the development drilling worth 33. Parameter M is the equivalent of DOcplex.CP Python API parameter `INTERVAL_MAX`. It represents a very large number used to set upper bounds to a value so large that it effectively equates to not setting any bound. Parameters O_t^L and O_t^U are bounds derived from the global objective of ore production for the mine. Each mine has its own objectives of ore production where the lower bound is usually the minimum feed necessary to keep the ore mill active. Parameters P_t^S and P_t^E define the start and end of each production period. The production periods are the periods over which the ore production objectives are defined. For example, a mine could define its minimal ore production to be of 3000 tonnes of ore per week, where O_t^L would step up weekly by 3000 and P_t^S and P_t^E represent the start and end of each week. Parameters R^M, R_l^L and R_v^V set different limits for different parts of the mine. These limits are often imposed by planners to avoid congestion in different zones or simply to avoid having too much fragmented rock to haul back to the surface for the mine capacity. Parameter $T_{sas'a'}^D$ represents the delay imposed in between two activities. These delays can be imposed for many reasons, including for example to wait for the drilling samples to be analyzed by the geology department in a gallery leading to stopes. Finally parameter T_s^{Max} imposes a maximum duration for the activities in one site. This is mostly due to ground stability reasons like in the stopes, where backfilling activities have to take place not too long after the haulage is completed.

A_c	:	Step function of the available crews of type c (%)
C_{sa}	:	Step function of the discounted cash flow associated with the activity a at site s with quarterly steps ($)
M	:	Large number representing the maximum possible number of time unit
O_t^L	:	Lower bound on the cumulative total tonnage of ore extracted at period t (tonne)
O_t^U	:	Upper bound on the cumulative total tonnage of ore extracted at period t (tonne)
P_t^S	:	Starting time unit of production period t
P_t^E	:	Ending time unit of production period t
Q_s	:	Rock tonnage in site s (tonne)
R_s	:	Rate of extraction at site s (tonne/shift)
R^M	:	Maximum possible total rate of extraction in the mine at any given time (tonne/shift)
R_l^L	:	Maximum possible total rate of extraction in level l at any given time (tonne/shift)
R_v^V	:	Maximum possible total rate of extraction in vein v at any given time (tonne/shift)
T^B	:	Backfill curing time to be respected in between adjacent stopes (shift)
$T_{sas'a'}^D$	:	Time delay imposed by planner in between activity a at site s and activity c' at site s' (shift)
T_s^{Max}	:	Maximum time span between the start of the first activity and the end of the last at site s (shift)
U_{sac}	:	Percentage of available crew type c required for activity a at site s

6.4.3 Variables

The model uses three kinds of variables. The first one, interval variables, is used to represent the activities. An interval variable has a size, a start and an end time and can be optional or not. An optional variable is a variable that can be absent from the final solution. The second type are sequence variables, that represent unordered sequences of interval variables over which one can impose special constraints. Sequence variables are used in this model to represent the relations between adjacent stopes. Finally, regular integer variables are used to represent variables quantity from the problem. In order to represent potentially fractional usage of crew, variable u_c is used as a percentage. For example, if the activities taking place at a given time require the work from 1.5 crew of type c, the value of u_c will be 150.

a_{sa}	:	Optional interval variable for the execution of activity a at site s with start time
$b_{sas'}$	:	Sequence variable linking variables a_{sa} and $a_{s'a'}$ $\forall s \in \mathcal{S}^B, c \in \mathcal{C}_s^L, s' \in \mathcal{S}_s^{Adj}, c' \in \mathcal{C}_{s'}$
q^O	:	Integer variable for the total tonnage of ore extracted
r^M	:	Integer variable for the total rate of extraction in the mine
r_l^L	:	Integer variable for the total rate of extraction in level l
r_v^V	:	Integer variable for the total rate of extraction in v
u_c	:	Integer variable for the percentage of available crew c being used

6.4.4 Objective

The objective of the model is to maximize the Net Present Value associated with the activities executed. The function `startEval` bellow simply evaluates the value of the discounted cash flow function C_{sa} for each activity a_{sa} at its start time.

$$Max \qquad \sum_s \sum_c \texttt{startEval}(C_{sa}, a_{sa}) \tag{6.1}$$

6.4.5 Constraints

Renewable Resources Constraints

One type of function and one type of constraints is used to constraint the renewable resources. The function $\texttt{pulse}(i, j)$ creates a step function with a step of height j for the duration of interval variable i. Constraint type $\texttt{alwaysIn}(i, j, k, l, m)$ forces a variable i, in the interval j to k, to take values in between l and m. Constraints 6.2 link variables u_c to the usage of each type of crew and Constraints 6.3 make sure that the amount required does not exceed the number available. Constraints 6.4 link variables r^M to the total rate of mining at all time in the mine and Constraints 6.5 limit this rate to its maximum value. Pairs of constraints

6.6 - 6.7 and 6.8 - 6.9 limit rates of extraction similarly but for levels and veins respectively.

$$u_c = \sum_s \sum_a \texttt{pulse}(a_{sa}, U_{sac}) \qquad \forall c \tag{6.2}$$

$$\texttt{alwaysIn}(u_c, 0, M, 0, A_c) \qquad \forall c \tag{6.3}$$

$$r^M = \sum_s \texttt{pulse}(a_{sa}, R_s) \qquad \forall a \in \mathcal{A}_s^H \tag{6.4}$$

$$\texttt{alwaysIn}(r^M, 0, M, 0, R^M) \qquad \forall c \tag{6.5}$$

$$r_l^L = \sum_s \texttt{pulse}(a_{sa}, R_s) \qquad \forall l, s \in \mathcal{S}_l, a \in \mathcal{A}_s^H \tag{6.6}$$

$$\texttt{alwaysIn}(r_l^L, 0, M, 0, R_l^L) \qquad \forall l \tag{6.7}$$

$$r_v^V = \sum_s \texttt{pulse}(a_{sa}, R_s) \qquad \forall v, s \in \mathcal{S}_v, a \in \mathcal{A}_s^H \tag{6.8}$$

$$\texttt{alwaysIn}(r_v^V, 0, M, 0, R_v^V) \qquad \forall v \tag{6.9}$$

Nonrenewable Resources Constraints

One new function is used to model the nonrenewable resources contraints. $\texttt{stepAtStart}(i, j)$ creates a step function with a step of height j at the start of interval variable i. Contrarily to `alwaysIn`, the value of the step is kept after the end of the interval variable. Constraints 6.10 and 6.11 assure that enough ore is extracted to keep the mill constantly fed. The decision to represent the ore feed as a nonrenewable resource with cumulative value along the time horizon comes from the fact that mines often use stock piles where ore is stored at the surface in waiting be processed at the mill. Hence, it is possible for a mine to extract more ore in a given period in order to compensate for smaller ore extraction in subsequent periods.

$$q^O = \sum_s \texttt{stepAtStart}(a_{sa}, Q_s) \qquad \forall s \in \mathcal{S}^O, a \in \mathcal{A}_s^H \tag{6.10}$$

$$\texttt{alwaysIn}(q^O, P_t^S, P_t^E, O_t^L, O_t^U) \qquad \forall t \tag{6.11}$$

Precedences Constraints

One new constraint type is used for the precedences constraints. The constraints $\texttt{endBeforeStart}(i, j, k)$ assure that interval variable i ends before interval variable j starts, with a minimum delay of k time units between them. Constraints 6.12 make the precedences links between the first activity of a site and the last of its predecessor while enforcing the required delay between the two activities. Constraints 6.13 make similar links but between predecessor and successor activities of the same site. Constraints 6.14 enforce the particular precedence relations between stopes linking the first activities from both stopes. The reason for this special link is that stopes precedences are not as strict as regular precedences. The point of these precedences is to give a general order to follow in the extraction of a vein's stope mostly for rock mechanics reasons. Only the first activities are linked together to represent the fact that when the first crew is done in a stope, it can start its work in the successor stope while the other crews finish theirs in the predecessor. Constraints 6.15 use the `endBeforeStart` formulation to assure that the maximum time span between all activities is respected. This particular formulation is recommended in [56] over one of the form `endOf`(j)-`startOf`$(i) < k$.

$$\texttt{endBeforeStart}(a_{s'a'}, a_{sa}, T^D_{sas'a'}) \qquad \forall s, a \in \mathcal{A}^F_s, s' \in \mathcal{P}_s, a' \in \mathcal{A}^L_{a'} \tag{6.12}$$

$$\texttt{endBeforeStart}(a_{sa'}, a_{sa}, T^D_{sasa'}) \qquad \forall s, a, a' \in \mathcal{A}^P_{sa} \tag{6.13}$$

$$\texttt{endBeforeStart}(a_{s'a'}, a_{sa}, 0) \qquad \forall s \in \mathcal{S}^{Stope}, a \in \mathcal{A}^F_s, s' \in \mathcal{P}^{Stope}_s, a' \in \mathcal{A}^F_{s'} \tag{6.14}$$

$$\texttt{endBeforeStart}(a_{sa'}, a_{sa}, -T^{Max}_s) \qquad \forall s, a \in \mathcal{A}^F_s, a' \in \mathcal{A}^L_{s'} \tag{6.15}$$

Backfilling Constraints

One last type of constraints is needed to represent the backfill constraints. `noOverlap`(i, j) constraints restrict all of the interval variables included in the sequence variable i not to overlap, with a minimal time delay of j between them. Constraints 6.16 assure that the curing time for backfill is respected between adjacent backfilled stopes.

$$\texttt{noOverlap}(b_{sas'}, T^B) \qquad \forall s \in \mathcal{S}^B, a \in \mathcal{A}^F_s, s' \in \mathcal{P}^{Adj}_s \tag{6.16}$$

6.5 Results

In order to test our model and compare it with a MIP formulation, we used the five datasets presented in [75]. These five datasets, based on data from a Canadian gold mine, represent five different planning scenarios with a number of possible activities between 842 for dataset 1 and 2229 for dataset 5 with ten different crew types involved. The starting time of all activities was limited to one year after the starting time. For the comparison purpose, the parameters and inputs for the model presented in [75] were modified for these five datasets in order to change the resolution from one week to one shift, so that both models can be equally compared over the datasets.

Tests for the CP and the MIP models were carried on the same computer with an Intel(R) Core(TM) i7-4770 CPU @ 3.40GHz and 16 GB of RAM. For the CP model, DOcplex.CP Python API using the Constraint Programming Optimizer of IBM ILOG CPLEX Optimization Studio 12.8.0.0 was used. For the MIP model, the Mathematical Programming Optimizer of IBM ILOG CPLEX Optimization Studio 12.8.0.0 was used. The relative gap tolerance was set to 0.01%, meaning that solutions 0.01% away from the best known upper bound were considered optimal and the time limit was set to 3600 seconds for both models.

Table 6.1 shows the result of the application of the CP model to the 5 different datasets. In the first section of the table, the objective value of the best solution found is displayed on the Objective line. The gap between this value and the best known upper bound is in the Gap line and the total number of feasible solution found in the branching process is displayed in the No of Solutions line. The second section of the table shows the time needed by the solver to reach solutions respectively 5 and 1% away from the optimal solution as well as the time at which the best solution was found and the total solving time. The differences between the times to find the best solutions and the total computation time are explained by the fact that in the majority of cases, the branching algorithm needs time to prove the optimality of a solution by lowering its upper bound after the discovery of the optimal solution.

Table 6.1 Constraints Programming model results for datasets 1 to 5

Solution	**D1**	**D2**	**D3**	**D4**	**D5**
Objective	9.99E+06	1.21E+07	1.59E+07	1.53e+07	1.52e+07
Gap (%)	0.1	0.1	0.1	0.1	0.1
No of Solutions	130	158	830	1084	611
Time (s)					
5%	23.5	13.2	420.7	1086.7	669.0
1%	24.5	31.5	422.1	1182.0	679.1
Best Solution	26.4	48.3	433.4	1240.4	774.3
Total	26.4	70.3	433.4	1252.0	1155.5

Table 6.2 show the results of the application of the MIP model to the five same datasets presented before. Once again, the value of the best solution found and the difference with the best known upper bound can be found respectively in the lines Objective and Gap. The LP Relaxation section shows the time needed to solve the linear relaxation to the problem and the value of its solution. The total computing time is displayed in the Total line in the last section.

From the results displayed in Table 6.1, one can see that the CP model proves to be very effective to solve the five problems. All of them were solved to the optimality limit with

Table 6.2 Mixed Integer Programing model results for datasets 1 to 5

Solution	**D1**	**D2**	**D3**	**D4**	**D5**
Objective	1.02E+06	-	-	-	-
Gap (%)	1240.5	-	-	-	-
LP Relaxation					
Time	514.7	862.7	-	-	-
Value	1.42E+07	2.08E+07	-	-	-
Time					
Total (s)	3600	2577.3	-	-	-

many feasible solutions found along the branching process. The times to reach the different % away from optimality also show that the optimality gap limit does not seem to have a large effect on the solving time; most of the good solutions found relatively close to the optimal solution. On the other hand, Table 6.2 clearly shows that the MIP formulation performs a lot worse than its CP equivalent. It could only find a feasible solution to the simplest scenario within the time limit of 3600 seconds. The solution found was very far from the optimality when compared to either the CP solution to the scenario (9.99E+06) or the gap to the upper bound found by the MIP branch and bound algorithm (1240.5%). All the other scenario filled the available memory, causing a memory error and halting the process before reaching the time limit. Only Scenario 2 could find the solution of the LP relaxation but in more than ten times what the CP model took to find the optimal solution.

6.6 Conclusion

The point of this article was to compare different formulations of the same problem, that is, the scheduling of short- and medium-term activities of an underground mine. The two models were based on different solving methods. The one presented in this article was based on the principle of CP, where its comparison was based on mathematical programming. The objective was to solve instances for up to a year ahead in order to allow for the long-term objectives to be considered. The results displayed in this article clearly show that the CP approach surpasses the MIP approach for all the scenarios tested. Of course, some assumption and approximation need to be made in order to model the problem as a pure CP problem, but so does the mixed integer approach. The level of precision reached by the model presented in this article (shifts planned for the next year) is probably too detailed for the underground mine reality, where planning often has to be redone weekly to address the many changes and unplanned events that happen daily. Considering that the computational time required to solve all instances are very low though, there is no reason not to plan with this precision for such a long time horizon. These results are also promising for longer-term planning models where time units could be extended in order to take into account time horizons of many years. Other than its computational results, the CP approach also offers advantages in an application context. The many constraints specific to each mine sites and mining techniques can be more easily modeled using the rich dictionary of CP function rather than having to use linear and integer variables. Of course, mixed integer constraints can ultimately model almost every situation but often to the expanse of readability and complexity. The dictionary of function makes the model more readable which makes it easier to maintain once implemented. From the promising results demonstrated for shorter-term planning in underground mines, the authors think that application of CP to these time horizons will be more and more present in the literature.

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CHAPITRE 7 DISCUSSION GÉNÉRALE

Afin de bien mettre en évidence les contributions de chaque chapitre, nous ferons ici un retour sur celle-ci. Dans le premier chapitre, la première contribution est de présenter un modèle permettant la planification précise des activités d'une mine souterraine. La seconde contribution est l'étude approfondie des propriétés des solutions, et plus particulièrement l'effet de la préemption sur celles-ci ainsi que sur la relaxation linéaire. La troisième contribution constitue la définition d'un cadre d'application du modèle dans une situation réelle ainsi que des résultats démontrant son efficacité.

Dans le cas du deuxième chapitre, la première contribution est de présenter un modèle de planification utilisant des variables de durée de grandeur différente afin de modéliser un problème de planification intégrée court et moyen terme. La deuxième contribution est de fournir un objectif alternatif qui permet d'améliorer la solution et les temps de résolution du problème et de démontrer la raison de cet effet par une analyse de la relaxation linéaire. La troisième contribution consiste à fournir un exemple d'application concrète du modèle ainsi que les manières de l'utiliser.

Dans le troisième chapitre, la première contribution est de présenter une formulation de programmation par contraintes pour un problème de planification minière, ce qui au mieux des connaissances de l'auteur, n'avait jamais été fait. La seconde contribution est de présenter une comparaison entre une approche en programmation en nombres entiers et une approche en programmation par contraintes au même problème de planification.

Finalement, le lecteur remarquera que le premier modèle permet une préemption complète, c'est-à-dire que les tâches peuvent être interrompues ou exécutées à un rythme variable, le second une préemption limité ou seul le rythme peut varier et le dernier ne permet techniquement pas de préemption. La raison du changement de la préemption complète à une préemption limitée est que dans les tests réalisés dans le premier article, il a été observé que le grand effet de la préemption sur la solution était en très grande partie due à la capacité du modèle ne pas nécessairement commencer une tâche au début d'une période de travail ou modifier le rythme d'exécution des tâches. Pour faciliter la résolution du deuxième modèle, il a donc été décidé de réduire les possibilités en ne permettant qu'une préemption limitée. Les endroits de travail utilisés pour tester le premier article ont cependant été sous-découpés en plus petite section afin de mitiger l'effet possible de l'impossibilité d'interrompre une tâche. Pour ce qui est du dernier modèle qui ne permet pas la préemption, la résolution est faite à une précision d'un quart de travail, soit la plus petite subdivision possible du travail en situation réelle. Ceci crée donc des possibilités de planification similaires à celles du deuxième modèle. La sous-division des endroits de travail utilisée est aussi la même que pour le deuxième article.

CHAPITRE 8 CONCLUSION ET RECOMMANDATIONS

Le corps des travaux ayant été présenté précédemment, nous ferons ici un bref retour sur les principales conclusions et présenterons les limitations propres à la solution proposée. Une ouverture sur les travaux en cours et futurs complétera le document.

8.1 Synthèse des travaux

La thèse présentée ici avait trois objectifs. Le premier était de développer un modèle permettant d'optimiser la planification à court terme des mines souterraines. Le second était de développer un modèle permettant l'optimisation intégrée des planifications à court et moyen termee et le dernier, de comparer les différentes approches. Pour ce faire, une revue de la littérature a été présentée pour démontrer l'absence de solutions existantes aux objectifs formulés. Un premier article a ensuite été introduit, présentant un modèle de planification des activités d'une mine souterraine pour le court terme. En plus de démontrer l'efficacité du modèle sur des données inspirées d'une mine réelle, l'article fournit une analyse détaillée de la relaxation linéaire du problème, une comparaison avec un modèle préemptif et un exemple d'application dans un contexte réel. Un deuxième article a ensuite présenté un modèle de programmation mathématique permettant la résolution du problème d'optimisation des planifications à court et moyen terme simultanément. En plus du modèle, cet article propose un objectif alternatif et démontre les raisons de son application, fournit un exemple d'application dans un contexte réel et analyse les différences de résultats de planification séparée et intégrée. Enfin, un troisième article présente un modèle de planification à court et moyen terme basé sur la programmation par contraintes. Ce dernier s'est montré très efficace à résoudre le problème abordé, offrant des résultats nettement meilleurs que les modèles précédents.

8.2 Limitations de la solution proposée

Bien que le dernier modèle présenté réponde très bien aux objectifs formulés, il présente tout de même certaines limitations. Tout d'abord, les données choisies pour effectuer les tests entrainent deux limitations. Premièrement, bien que les modèles utilisent les deux méthodes de minage les plus populaires dans les mines de métaux québécoises, il en existe beaucoup d'autres qui peuvent avoir leurs contraintes particulières. Même s'il s'agissait sans doute de changements mineurs, les modèles devraient certainement être modifiés dans le cas où une nouvelle méthode de minage serait utilisée. Deuxièmement, les données utilisées représentent une opération de petite à moyenne taille. Ainsi, l'effet sur une opération de grande taille impliquant encore plus de ressources reste inconnu.

Une autre source de limitation vient du mode de transport de la roche fragmentée. La première concerne l'utilisation de monteries pour le transport du minerai, comme c'est le cas pour nos données. Les monteries sont des tunnels fortement inclinés entre les niveaux servant à acheminer la roche des différents points d'extraction jusqu'au puits, pour y être remonté. Ces monteries ayant des capacités limitées, lorsqu'elles ne sont pas vidées simultanément par le niveau du bas, elles ne peuvent être remplies indéfiniment par les chargeuses-navettes du niveau supérieur. Cette contrainte est partiellement prise en compte par les limites de capacité de niveaux et de veines incluses dans les modèles, mais ne sont que des approximations laissées au planificateur. Deuxièmement, dans le cas où le minerai serait remonté à la surface par des camions, on pourrait ajouter une ressource au modèle pour représenter cette nouvelle ressource.

D'autres limitations viennent de la modélisation des ressources. Premièrement, dans les données utilisées une seule équipe de mineurs était responsable de toutes les tâches de développement, à l'exception du déblaiement et du nettoyage. Ces tâches ont donc été modélisées comme une seule ressource. Dans une mine où ce ne serait pas le cas, il faudrait créer de nouvelles ressources pour les équipes responsables de l'exécution de chacune de ces tâches. Deuxièmement, les ressources étant limitées par une capacité globale plutôt qu'individuelle, le modèle ne permet pas de déterminer précisément quelles tâches seront attribuées à quelles équipes et donc de prendre en compte les temps de déplacement entre les différents endroits de travail.

8.3 Améliorations futures

Les résultats de recherche présentés ici ne sont pas une fin en soi. Que ce soit pour adresser les limitations mentionnées dans la section 8.2 ou pour améliorer la capacité des modèles, beaucoup de travail reste à faire dans le domaine de l'optimisation minière souterraine. Parmi les travaux s'inscrivant dans la succession de cette recherche, une tentative de modification du modèle est en cours afin de vérifier son applicabilité à un problème de mine en fosse.

Pour ce qui est du modèle présenté dans cette thèse, l'inclusion de l'aspect stochastique de la planificaiton directement dans le modèle fait partie des avenues de recherche à explorer. Finalement, une application réelle et les commentaires de planificateurs sur leur utilisation du modèle contribueraient certainement à améliorer davantage le modèle. Une autre avenue possible pourrait être d'étendre le modèle à la planification en temps réel, en tenant en compte l'attribution de ressources spécifiques à des tâches et incluant les temps de déplacement entre les différents endroits de travail. Ceci devrait probablement être résolu dans un premier temps dans un modèle séparé utilisant les solutions des modèles développés précédemment comme paramètre et éventuellement, une approche intégrée pourrait être considérée.

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